Firm Dynamics and the Labor Market Effects of Rural-Urban Migration

Clément Imbert Gabriel Ulyssea*

August 10, 2022

Abstract

This paper investigates the labor market effects of rural-urban migration in Brazil, and how they are shaped by firm dynamics. We instrument immigration in each city by a shift-share instrument based on origin-level push shocks. We find that internal immigration has no effects on unemployment, it induces a reallocation of wage employment from informal to formal jobs, while decreasing wages in both the formal and informal sectors. These effects can be explained by formal firm dynamics: higher immigration rates lead to increased entry in the formal sector, a larger number of formal firms and formal jobs. To rationalize these results, we develop and estimate a model of firm dynamics and informality, and simulate the effects of an immigration labor supply shock. In line with the IV results, we find that there is an increase in formal firms and jobs, with a relative growth of low- and medium-productivity firms. These effects are driven by formalization of informal firms, and not by the creation of new formal firms. Aggregate output increases, but productivity per worker falls. These dividends would be larger under stricter enforcement and lower informality.

*Imbert: University of Warwick, c.imbert@warwick.ac.uk. Ulyssea: University College London, g.ulyssea@ucl.ac.uk. We thank conference and seminar participants at ETH-Zurich, UAB, STEG, TSE, UCL, LISER, CEPR, MadMac and UPF for helpful comments. We are grateful to David Crommelynck, Henrique Mota, Ana Beatriz Pousada and Viktor Veterinarov for outstanding research assistance throughout different stages of this project. Ulyssea gratefully acknowledges the generous funding from the ERC Starting Grant, project number 852744, acronym FORCEDMIGDEV.

1 Introduction

Urban population is growing fast in developing countries: it grew 12.5 percent in 2015–2000, with a 64.7 percent growth projected until 2050 (UNCTAD, 2021). Whether urban economies will be able to generate enough good jobs to accommodate these new workers is a key question for economic development (The World Bank, 2012; Alfonsi et al., 2020). Indeed, developing countries are characterized by low firm growth (e.g Hsieh and Klenow, 2014), high shares of informality, underemployment and unemployment, especially among young workers (Bandiera et al., 2021). Moreover, rural-urban migration still accounts for a substantial fraction of population growth in urban areas (Jedwab et al., 2017). The dominant view that stems from the works of Harris and Todaro (1970) and Fields (1975) predicts that rural migrants join the urban pool of unemployed or informal workers, queueing for a limited number of formal jobs whose wages are fixed by minimum wage laws. Consistent with this view, recent evidence shows that rural-urban migration increases informality and unemployment in the short-run (El Badaoui et al., 2017; Kleemans and Magruder, 2018).

The "Harris-Todaro view", however, relies on strong assumptions that do not match some key features of urban labor markets in developing countries. First, labor markets are not dual: formal and informal firms coexist within narrowly defined industries, their productivity distributions overlap, and a large fraction of informal employment is located within formal firms (Ulyssea, 2020). Second, wages in the formal sector adjust even in the presence of binding minimum wages: Imbert et al. (2022) show that an inflow of rural migrant workers can slow down wage growth and increase employment in large formal manufacturing firms in China. Crucially, this view ignores firm dynamics and the fact that labor supply shocks induced by immigration can have first order effects on firm entry and growth.

This paper investigates the labor market and aggregate effects of rural-urban migration in local urban economies in Brazil. We do so in three steps. First, we use a shift-share IV design to identify the causal effects of immigration on local labor market outcomes and formal firm dynamics at destination. We combine matched employer-employee data on the universe of formal firms and workers in Brazil between 1995 and 2018, with individual level data from two Demographic Censuses (2000 and 2010). To construct our shift-share instrument, we use detailed information on previous migration patterns across municipalities (the shares), and granular

data on agricultural suitability and land use combined with international price shocks to construct push-shocks at origin (the shifts). Second, to rationalize our IV results, we develop a new model of firm dynamics that features the intensive and extensive margins of informality (Ulyssea, 2018), heterogeneous growth profiles, endogenous entry and exit. Third, we estimate the model and use it to perform counterfactual simulations to shed further light on mechanisms, and to assess general equilibrium effects on outcomes such as aggregate productivity and output.

We first use our shift-share IV design to estimate the local labor market effects of immigration at urban destinations. Surprisingly, we find that internal immigration in Brazil between 2000 and 2010 not only had no effect on unemployment but instead increased the share of formal employment: an increase in the immigration rate of one percentage point increases the share of workers in formal wage employment by 0.4 percentage points (a 2.5 percent increase). This effect is entirely driven by a shift of workers from informal into formal jobs, with no change in wage employment overall, unemployment, or self-employment. Wages in the formal and informal sectors fall by 2.1 and 1.9 percent, respectively. These immigration flows do not change the skill composition at destination, which suggests that our effects are not driven by changes in workforce skill composition. Consistently, we find similar results if we focus on residents only, and for high- and low-skilled workers.

We then turn to the effects of internal immigration on formal firm outcomes. We find that an increase in the immigration rate by one percentage point leads to a 2.4 percent increase in the number of firms and a 2.2 percent increase in the number of formal jobs. The effects are driven by a substantial increase in firm entry, accompanied by an increase in firm exit of smaller magnitude. There are no effects on average firm size nor firm growth, and we show that these new firms are not being created by the migrants themselves. When we examine dynamic effects, we show that these effects remain constant or slightly increase throughout the 2010's (until 2018). These results are the mirror image of the mechanisms emphasized by the recent literature that highlights the labor supply slowdown as the main explanation for the "start-up deficit" in the U.S. (Pugsley and Sahin, 2019; Karahan et al., 2019; Peters and Walsh, 2022).

We carry out a number of robustness checks to alleviate concerns about identification. Reassuringly, our results are unaffected once we include pre-trends in outcomes between 1997-98 as controls. One might still worry that the instrument may be cor-

related with omitted variables which could all have an independent effect on firm and worker outcomes. We show that our results are robust to controlling for past trends in outcomes, for GDP and population in 2000, for the share of manufacturing and construction in 2000, and for immigration between 1995 and 2000. Another concern may be that the price shocks in rural areas may affect firms through other channels than migration. We check that our results are robust to excluding agricultural firms or firms that process agricultural products, to controlling for shocks to agricultural prices that could depress local demand for firms' products, and to controlling for exposure to capital reallocation from rural areas via the network of banks' branches across municipalities.

As a final robustness check, we use an alternative identification strategy that uses droughts as push shocks combined with pre-existing migration networks to instrument for immigration. We find very similar effects on both workers and firms. Even though the incidence of these two push shocks (prices and droughts) is correlated, it is far from a perfect overlap. This means that our results are not particular to a specific set of compliers from specific pairs of sending and receiving regions. The composition of migrants is also not significantly affected by the type of push shock used: both shocks generate immigration flows that do not change skill composition at destination but which lead to a larger proportion of male and young workers. Given that this is also the typical profile of the growing urban workforce in many developing countries (e.g. Bandiera et al., 2021), we believe our results are relevant to different contexts.

We then turn to the effects on the composition of firms and jobs by productivity levels, which we proxy by residual firm-level average wage. We show that immigration leads to a reallocation of firms and jobs towards the middle and bottom quartiles of the firm productivity distribution, and away from the top quartile. These reallocation effects are again largely driven by increased entry rather than greater survival or expansion of existing firms. This could be expected if greater entry is a consequence of formalization of existing informal firms, which are on average less productive than formal firms. We cannot directly assess this conjecture with the data, but we do so in the counterfactual analysis using the structural model.

The composition of firms and jobs also shifts towards smaller firms, and services, retail and construction, while moving away from manufacturing. These sectoral composition effects are intuitive, as construction is a sector that traditionally employs many migrants, while retail and services are more labor intensive and require lower

and less specific skills than manufacturing. Non-tradables are also more likely to benefit from the positive demand shock brought about higher presence of immigrants.

The vast economic heterogeneity across regions in Brazil allows us to investigate two important sources of heterogeneous effects. First, we use the enforcement capacity measure used by Ponczek and Ulyssea (2022) to split the sample between municipalities closer or further away from labor offices in charge of enforcing labor regulations on formal firms. The results are driven by cities with higher enforcement capacity (and potentially stricter enforcement), even after conditioning for different potential confounders. If one thinks of informality as an expression of broader frictions and obstacles to firm growth, or as a size dependent distortion itself (e.g. Ulyssea, 2018), then these results imply that the larger the frictions the lower the dividends to urban economies from a growing labor force.

Second, we examine heterogeneous effects across terciles of GDP per capita at baseline (in 2000). Brazilian regions exhibit large disparities in economic development: average GDP p.c. in the top tercile is more than twice as large the average in the middle tercile and 4.5 times larger than the bottom one. Thus, these results capture heterogenous effects across very different parts of the development spectrum. We show that the effects on workers and firms are concentrated in municipalities in the bottom and middle terciles, which has two important implications. First, the heterogeneity by enforcement levels is not simply capturing variation in economic development. Second and more broadly, it indicates that urban economies in the lower end of the development spectrum are the ones that benefit the most from the inflows of migrants.

To rationalize these surprising findings, we extend the canonical model of firm dynamics of Hopenhayn (1992) to allow for heterogeneous growth profiles (similar in spirit to Sterk et al., 2021), and that includes the two margins of informality considered in Ulyssea (2018): (i) whether to register their business or not, the extensive margin; and (ii) whether firms that are formally registered hire their workers with or without a formal contract, the intensive margin. Given that informal employment is often the main port of entry at destination for many migrants, especially low-skill ones, to carefully model informality is arguably important. In particular, the intensive margin immediately implies a direct link between migration shocks and formal firms. Even if the bulk of these migrants ends up in informal jobs, this can have important direct effects on formal firms and workers.

A second key element of the model is the presence of heterogeneous growth profiles across firms. This implies that both margins can have opposing effects in the economy. On the one hand, informality can act as a stepping-stone for potentially high-growth firms. More broadly, it introduces greater *de facto* flexibility that could lead to higher firm entry and growth, which would otherwise not happen due to other frictions, such as burdensome regulations. However, it also allows less productive firms to survive, thus weakening the natural selection process in the economy. It may also allow these firms to compete with more productive formal firms, thus shifting resources away from high-productivity firms and hindering their ability to grow. The net effect of these forces and their interaction with immigration shocks is an empirical question.

In the third and final part of our analysis, we structurally estimate the model and use it to perform counterfactual simulations of the effects of a once-and-for-all 10 percent labor supply increase, which corresponds to the 80th percentile of the migration shocks we see in the data. In order to further investigate the role of the informal sector, we analyze this labor supply shock under two different scenarios: (i) the baseline Brazilian economy in 2003; and (ii) a scenario where enforcement along both margins of informality is 50 percent higher than the baseline.[1]

The counterfactual results confirm our empirical analysis: the supply shock leads to a small reduction in the share of informal workers and a sizable increase in the number of formal firms. This increase in the number of formal firms is completely explained by higher formalization of informal firms throughout their life cycle, rather than entry of newly created formal firms. Wages fall, as expected, but the elasticity is far from one, which indicates that the increase in labor supply is being partially met by an increase in production and therefore in the demand for labor. Both output and income increase, but by less than the increase in labor supply, which implies that output and income per capita are falling. These effects are rationalized by the composition effects that show that the share of firms and jobs increases at the middle bottom of the productivity distribution due to the formalization of less productive informal firms. This is in contrast with the results of Peters (2022) in the context of post-war West Germany and highlights the importance of accounting for developing countries' specificities, in particular the presence of large informal sectors. Indeed, our results show that these reallocation effects are largely improved under stricter

[1]This is a large but plausible variation in enforcement, which is consistent with the variation in enforcement levels across regions in Brazil (Ponczek and Ulyssea, 2022).

enforcement, which translates into larger effects on total output and income, although productivity per worker still declines.

Our paper is relevant to three strands of the literature. First, we bring new evidence on rural-urban migration and urban labor markets in developing countries. The literature has grown around the idea of wage rigidities in the formal sector and the prevalence of urban unemployment or underemployment in the informal sector (Harris and Todaro, 1970; Fields, 1975). In this setting, rural-urban migration only adds workers to the pool of unemployed or informal workers queueing for a limited number of formal jobs. Consistent with this view, the empirical literature tends to find negative employment effects for resident workers, especially in the formal sector (Kleemans and Magruder, 2018; El Badaoui et al., 2017; Corbi et al., 2021).[2] In contrast with these short-run results, we document positive effects on resident formal employment in the longer-run, which we argue are linked to firm dynamics.

Second, we contribute to the literature on population growth and firm dynamics. Our findings that internal migrant labor supply spurs formal firm and job creation in Brazil are a mirror image of the recent literature on demographic decline and the "start-up deficit" in the US (Pugsley and Sahin, 2019; Karahan et al., 2019; Peters and Walsh, 2022). The US literature argues that the demographic slowdown had negative effects on firm entry, with detrimental consequences for labor reallocation, employment and firm growth (e.g. Decker et al., 2014; Pugsley and Sahin, 2019). We focus on labor supply growth induced by internal migration, emphasizing the role the two margins of informality, which is key for understanding labor supply effects on firm dynamics in developing countries. We bring to this literature some of the first empirical evidence on the effect of exogenous increases in internal migrant labor supply on formal firms dynamics and worker allocation between the informal and formal sectors in a developing country context.

Finally, our paper relates to the growing literature on immigration and firms (Lewis, 2011; Peri, 2012; Olney, 2013; Dustmann and Glitz, 2015; Kerr et al., 2015; Mitaritonna et al., 2017). While most authors focus on incumbent firms or aggregate outcomes, Dustmann and Glitz (2015) highlight the role of firm entry in the absorption of immigrant labor supply. A handful of papers is devoted to developing country contexts. Imbert et al. (2022) study large-scale manufacturing firms in China and

[2]These effects are typically larger in magnitude than in the literature on international migration in developed countries (Card and DiNardo, 2000; Card, 2001; Borjas, 2003; Clemens and Hunt, 2019)

show that internal immigration linked to agricultural price shocks leads to employment growth and a shift towards labor-intensive production patterns. Albert et al. (2021) use the same census and firm data as we do to study the effect of climate change on capital and labor reallocation in Brazil. Our contribution is to focus on aggregate labor market effects, the role of informality and firm dynamics in shaping these effects. Importantly, the capital reallocation channel cannot explain our results: in our robustness analysis we directly control for the exposure to capital reallocation via the network of banks' branches and our results remain unaltered. By combining both reduced-form and structural analyses, we are able to speak to aggregate effects of these migration flows at destination. In particular, we show that rural migrants generate economic growth in urban areas but decrease productivity per worker. This is consistent with the view that the reallocation of workers across sectors increases aggregate output and narrows the rural-urban productivity gap (Gollin et al., 2014).[3]

The paper is structured as follows. Section 2 presents the data and Section 3 the instrumental variable results. Section 4 develops a theoretical model, which is estimated in Section 5, which also presents counterfactual analysis.

2 Data

This section describes the datasets we use in the next sections to estimate the causal effect of immigration on the local economy (Section 3) and to structurally estimate our model (Section 5). We also provide descriptive statistics on workers and firms in our data.

Migration and Labor Market Outcomes

The first dataset we use is the Decennial Population Census, which contains information on individuals' socioeconomic characteristics and labour market outcomes. Crucially, the Census also contains detailed information about individuals' migration patterns. In our analysis, we focus on the last two waves of the Census – 2000 and

[3]Another channel through which immigration may affect employment growth is through research and development. In the US context, the literature has documented the positive contribution of high-skilled immigration to innovation and science (Moser et al., 2014; Akcigit et al., 2016). In Brazil, (Bustos et al., 2018) find that research and innovation decline when agriculture releases low-skilled workers.

2010 –, and restrict the sample to working-age adults (15 to 64 years old).

Our geographical unit of analysis is a Minimum Comparable Area (MCA), which combines municipalities whose borders have changed during the study period: there are 3,658 such MCAs. We will call them "municipalities" for simplicity. We define as migrant a person who came to their current municipality of residence in the last ten years, and compute the cumulative immigration flows between 2000 and 2010 between each municipality pair. We focus on flows to urban locations, as defined in the census (88% of all migration between 2000 and 2010), and across state borders (40% of migration to urban areas between 2000 and 2010). We then compute the immigration rate in each urban destination as the sum of immigration flows over the decade divided by the population in 2000. This is our main endogenous regressor in the instrumental variable analysis (Section 3). Internal immigration in the average urban destination is large 19.7% overall, 6.3% for state-to-state migration (Panel D in Table 1). Figure 1 shows the geographical variation: cities with high immigration rates are either the coastal megacities or smaller cities located in booming agricultural regions in the center of the country.

We also use the census to compute socio-demographic characteristics and labor market outcomes for each destination municipality, which are shown in Table 1. The main socio-demographics (Panel A) are the share of female, of young (below 18) and high-skilled (completed secondary education and above) in the working age population. Our main labor market outcomes are the share of working-age adults employed in the private sector as their main occupation, which we split between formal and informal sector work. The Brazilian census asks workers whether they have a "signed work booklet" (*carteira de trabalho assinada*): we categorize as formal workers those who have the booklet, and the others as informal workers.[4] We also compute the average real hourly wage earned by private sector workers by dividing their monthly income from their main occupation by the hours they worked in the last week multiplied by 4.33. We do this for all private sector workers, and separately for formal and informal workers. As Panel B in Table 1 shows, formal private wage work expanded in Brazil between 2000 and 2010, while informal wage work stagnated. Wages increased by 18% (17% in the formal sector).

[4]See for example, Ponczek and Ulyssea (2022) for a description of labor regulations in Brazil.

Firms

The second data set used is the *Relação Anual de Informações Sociais* (RAIS), which is a matched employer-employee administrative dataset from the Ministry of Labour in Brazil. It contains the universe of formal firms and workers between 1997 to 2018. We use these data both in the instrumental variable and the structural estimation.

For the instrumental variable analysis, we use the RAIS data to compute municipality-level moments related to firm dynamics (Panel C in Table 1). We compute the log of the total number of formal firms, and of formal jobs in December 2000 and 2010. We also compute the log of entrants, i.e. the log of the number of firms that enter the sample between 1999 and 2000, and the log of exiting firms in 2000, i.e. the log of the number of firms that exit the sample between 1999 and 2000. Entry in the data can be due to firm creation or formalization of an establishment which was previously operating informally. Exit from the data is likely due to establishment closure, as few formal firms can go under the radar. We also compute the real average wage in each municipality by dividing the monthly wage by the number of hours worked and by an inflation index and averaging it over all formal employees. As Panel C in Table 1 shows, between 2000 and 2010 Brazil has seen a tremendous growth in the number of formal firms (40%) and formal workers (60%).

To estimate the model, we need additional data on informal firms and informal workers within formal firms to compute the relevant moments to be combined with the Simulated Method of Moments estimator. We thus complement the RAIS data with the ECINF survey (*Pesquisa de Economia Informal Urbana*), a cross-section representative of all Brazilian firms with up to five employees, which was collected by the Brazilian Bureau of Statistics (IBGE) in 2003.[5] This is a matched employer-employee data set that contains information on entrepreneurs, their businesses and employees. In particular, it includes information about the businesses' age (months/years in operation), their formalization status and of their employees. Thus, the data allow us to directly measure both the extensive and the intensive margins of informality, i.e. establishments which are informal and workers who are informally employed, including by formal firms.[6]

[5]The actual survey data include firms with up to 10 employees, but the information for firms with more than five employees is not representative at the national level (see de Paula and Scheinkman, 2010, for a detailed description of the ECINF data set).

[6]The ECINF data is not representative at finer levels of geographic disaggregation (only at the

Push Shocks

In order to identify the causal effect of migration on destination outcomes, we need variation in migration flows that are exogenous to conditions at destination. For this, we construct two origin-level push shocks: in our main specification, we consider variations in agricultural income due to changes in international crop prices, and as a robustness exercise, we use the occurrence of droughts.

We use three additional data sources to construct these shocks. For the price shock, we collected monthly information on international prices for 12 crops (including bananas, cocoa, coffee, cotton, maize, orange, rice, soybeans, sugar, tobacco, wheat and wood) between 1970 and 2011. For each crop c and month m, we compute a price shock ε_{cm} as the residual of an AR(1) process. We then aggregate these crop- and month-level shocks into a (origin) municipality level shock s_o^p using the share of each crop π_{oc} in the value of agricultural production in the municipality of origin o. We use the 1980 Agricultural Census to compute these shares for each municipality and crop. The construction of the shock is similar to Imbert et al. (2022) who study rural-urban migration in China. Figure 2 shows the geographic distribution of the price shock. Formally, the price shock is given by:

$$s_o^{prices} = \sum_m \sum_c (\pi_{oc} \times \varepsilon_{cm}) \tag{1}$$

For the drought shock, we combine a measure of dryness in each municipality in each month, the SPEI (Standardized Precipitation-Evapotranspiration Index),[7] with information on the growing season of each crop and the value of each crop harvested in each municipality from the 1980 Agricultural Census (see Appendix C for more details on the drought shock).

state level), which prevents us from using it in the instrumental variable analysis.

[7]The SPEI has been built by climate scientists Vicente-Serrano et al. (2010) and can be freely downloaded here https://spei.csic.es/home.html. It has been used in economics by Bertoli et al. (2020) and Albert et al. (2021).

3 Instrumental Variable Analysis

3.1 Empirical Design

We estimate the impact of immigration on labor markets outcomes by regressing the change in outcome y in municipality d between 2000 and 2010, on the immigration rate over the decade Mig_d:

$$\Delta y_d = \beta_0 + \beta_1 Mig_d + \beta_2' \mathbf{X_d} + u_d \tag{2}$$

where $\Delta y_d = y_{d,2010} - y_{d,2000}$ is the long difference in labor market outcomes: employment rate in the private wage sector (formal and informal) and log wages in the private sector (formal and informal). The vector $\mathbf{X_d}$ includes socio-demographic controls: the share of female, young and high-skilled workers in municipality d in 2000. We report robust standard errors, and weight regressions by population in 2000.

We then turn to firm outcomes, for which we have annual data between 1995 and 2018. We use a specification similar to 2 and regress changes in the number of formal firms, in the entry and exit rates, in the number of formal jobs and in the firm wage on the migration rate, controlling for socio-demographic controls. To ensure the stability of the entry and exit estimates, we compute outcomes as two-year averages. For comparability with the labor market results, we first focus on the decadal changes between 1999-00 and 2011-12. We then exploit the full extent of the panel data and estimate dynamic effects of migration on changes in outcomes between 1999-00 and 2009-10, 2011-12, 2013-14, 2015-16 and 2017-18. We also go back in time to investigate the presence of pre-trends, and use as a dependent variable the change in outcomes between 1997-98 and 1999-00.

Identification

Since we estimate all regressions in first differences, our specification implicitly accounts for municipality fixed effects. By including controls, we also allow for differential trends across municipalities with different initial socio-demographic conditions. However, the regressions above may not identify the causal effect of immigration on labor markets and firms due to reverse causation and omitted variable bias. For example, cities with thriving labor markets are likely to attract more migrants.

To overcome these threats to identification, we rely on a shift-share instrumental variable design, which has been extensively used in the literature (e.g. Borusyak et al., 2018). More specifically, we rely on cross-sectional variation across destination municipalities in their pre-existing migration networks with different origins (the "share"), and time variation in push shocks that affect migration incentives at origin (the "shift"). Formally, the instrument writes:

$$Z_d = \sum_o \lambda_{o,d} s_o \tag{3}$$

where Z_d denotes the instrument for immigration into the municipality of destination d, based on the price shock s_o to agricultural productivity in the municipality of origin o described in section 2. The $\lambda_{o,d}$ denotes the share of migrants from origin o among migrants who had come at destination d between 1995 and 2000.[8] We then use Z_d as an instrument for Mig_d in a 2SLS estimator.

There are a number of threats to our empirical strategy. Regarding identification, one may worry that shifts are not randomly assigned, but correlated with potential outcomes at destination. To alleviate this concern, we carry out placebo checks in which we regress changes in firm outcomes between 1997-98 and 1999-00 on immigration between 2000 and 2010. We also estimate our main specification including as controls lagged changes in outcomes, log GDP at baseline, and shares of the different industries. Another concern is that immigration between 1995 and 2000, which we use to compute the shares, may have long lasting effects on firm outcomes between 2000 and 2010. To alleviate this concern, we add 2000 log population and the 1995-2000 migration rate as controls to our main specification.

There may also be a number of concerns about the exclusion restriction needed for the instrument to be valid. First, shocks to rural incomes may affect the demand for goods produced by urban firms (Santangelo, 2016). To control for this potential demand channel, we control for local agricultural prices in the municipality of destination. Second, negative shocks to rural productivity may lead to a reallocation of capital towards other parts of the country, including migrants' destinations (Albert et al., 2021). To account for this channel, we control for the exposure of each destination via the bank network, measured as the share of loans that come from banks that

[8] The 2000 Census only contains retrospective information on migration going 5 years back, hence we compute the migration shares from 1995 to 2000.

draw more deposit from municipalities hit by price shocks. Third, international price shocks for agricultural commodities may have indirect effects on urban firms that process agricultural goods. To respond to this concern, we exclude from the sample agricultural firms, and manufacturing firms that process food and beverages or tobacco. Finally, we use an alternative shift-share instrument based on the prevalence of droughts in each municipality of origin and check that we obtain similar results.

3.2 Effects on Labor Markets

We start by discussing the effects of internal immigration on urban labor markets. Table 2 presents the results. The OLS estimates suggest that immigration is associated with higher formal employment, lower informal employment and (insignificantly) higher wages. One would expect these results to be biased since migrants would be attracted by destinations with better labor markets outcomes. We next turn to the IV estimates which have a more causal interpretation. Surprisingly, the effect on formal employment is even more positive, and the effect on informal employment more negative than what the OLS estimates suggest (Columns 2 and 3). A one percentage point increase in the migration rate increases formal employment by 0.40 percentage points (a 1.7% increase from the mean of 23%) and decreases informal employment by 0.29 percentage point (a 2.9% decrease from the mean of 10%). Overall, wage employment increases by an insignificant 0.10 percentage points: most of the increase in formal sector work is driven by a shift away from informal employment. In Appendix Table B.6 we show that there is no effect on non-employment, self-employment, or domestic work, but a small decline in the fraction of workers who report being employers and working in the public sector. Turning to Columns 4 to 7, we find strong evidence that immigration decreases wages. A one percentage point increase in the immigration rate reduces wages in the formal sector by 2.1%, by 1.9% in the informal sector. The overall effect on wages is smaller, 1.6%, which is due to a compositional effect, as workers shift from the low paying informal sector to the formal sector.[9]

Our finding that internal immigration increases formal employment, with no effect on unemployment runs counter the idea that internal migrants join the ranks of casually employed or unemployed workers in urban areas, a view that dates back at least

[9]Although we use the main occupation in the last month and monthly wages as our main outcomes, we check in Appendix Table B.5 that the results are similar when we use the share of hours spent in each occupation and hourly wages instead.

to Harris and Todaro (1970) and Fields (1975), and which has been supported by recent empirical evidence (Kleemans and Magruder, 2018; El Badaoui et al., 2017). Our finding that immigration decreases formal as well as informal wages also contradicts the view that formal sector wages do not adjust to labor supply shocks, due for example to binding minimum wages. The difference is not merely due to the specificities of the Brazilian context: minimum wage regulations are implemented in Brazil and the real minimum wage more than doubled between 1996 and 2018 (Engbom and Moser, 2021). One potential explanation for this difference is that the other empirical papers estimate short-run effect on employment (typically one year after the shock), while we consider changes in employment over a decade.[10]

The estimated negative effect of immigration on wages overall are consistent with the literature on internal migration in developing countries (Kleemans and Magruder, 2018; El Badaoui et al., 2017), which tends to find larger negative effects than the literature on international immigration in developed countries (Card, 2001; Borjas, 2003). The literature has also emphasized the importance of looking at the labor market response to immigration separately by skill, especially when migrants have different skill levels than natives (Dustmann and Glitz, 2015). In the case of Brazil in the study period, interstate migrants are not less skilled than local residents, so that we would not expect immigration to change the skill composition of the workforce. We check in Appendix Table B.3 that our results are unchanged when we include only resident workers and exclude migrants from the sample. Appendix Table B.4 presents very similar effects on employment and wages for high- and low-skilled workers, although wage effects are more negative for low-skilled workers.

3.3 Effects on Firm Dynamics

Our results from the population census suggest that internal immigration decreases labor costs, which allows the formal sector to grow. We now turn to the firm side. We start by examining the effects of immigration on decadal changes of the number of formal firms, entry, exit, number of jobs and firm average wages. Panel A in Table 3 present the OLS estimates, while Panel B presents the IV results. Comparing the OLS and IV results, the effects on number of firms, entry, exit and number of

[10] Imbert et al. (2022) use five-years differences and find similar negative wage effects and positive employment effects of internal immigration on large manufacturing firms in China.

jobs remain positive and increase in magnitude. Our estimates indicate that a one percentage point increase in the immigration rate – which corresponds to 14.5 percent of a standard deviation – leads to an increase of 2.4 percent in the number of firms and 2.2 percent increase in the number of formal jobs. This is entirely driven by the substantial increase in entry, by 7.5 percentage points. Exit also increases, which is expected given the large increase in firm entry.

To put these effects in perspective, the start-up deficit (i.e. decline in entry rate) documented in the U.S. corresponds to a decline of 5 percentage points between 1980 and 2012 (Pugsley and Sahin, 2019). Hence, the labor supply shock induced by rural to urban migration has a first order effect on firm dynamics, which operates via an increase in start-up rates. The negative effect on average firm wage (in the price shock) is consistent with the worker results presented in the previous section.

We can also use the annual nature of the RAIS firm data to analyze the dynamics of these migration effects. Specifically, we estimate the effects of migration on changes in outcomes for different time windows: between 1999-00 and 2009-10, 2011-12 (our main results), 2013-14, 2015-16, and 2017-18. As Figure 3 shows, the effects tend to be somewhat stable or slightly increasing over time.

3.4 Robustness

The main threat to our identification strategy would be a failure of the common trend assumption, i.e. if firms or workers located in destinations that receive immigration shocks between 2000 and 2010 would have experienced differential changes in outcomes even in absence of the shocks. The shift-share instrument we use partly alleviate this concern: in our case, identification relies on the assumption that although the baseline migration shares may be endogenous to future outcomes, the shifts (push shocks) are as good as randomly assigned (Borusyak et al., 2018; Adão et al., 2019).

To alleviate concerns that pre-trends in outcomes may drive our results, we perform a number of robustness checks presented in Appendix Section C and Tables C.2 to C.4. We first include lagged changes in outcomes in Panel A, and we also control for log population (Panel B). Another potential threat to our instrumental variable strategy would arise if migration rates are very persistent overtime, so that we are capturing the effects of previous migration waves, a concern which has been raised in particular by Jaeger et al. (2018). To alleviate this concern, we control for migration

rates between 1995 and 2000 in our main specification, in Panel C. Finally, we also control for log GDP (Panel D) and industry composition (Panel E), all computed at the beginning of the study period in 2000. Our results are robust to these controls.

Although we interpret our IV results as the effects of immigration, there are a number of additional channels through which agricultural prices shocks could affect urban areas. First, we test that our results do not change after we control for agricultural price shocks in the municipality of destination, which may affect demand for good produced by local firms (Panel A in Appendix Tables C.3 and C.4). Second, we control for indirect exposure to price shocks at origin via the bank network, and check that our results are not driven by capital reallocation towards the firms at destination (Panel B in Appendix Tables C.3 and C.4). Finally, we check that our results are not driven by firms that produce agricultural goods or process them: our results are unchanged when we exclude them (Panel C in Appendix Table C.3).

As a final robustness check, we estimate the effects of migrant labor supply using a different shift-share instrument based on droughts at origin (see Section 2). As Table C.6 shows, we find similar positive effects of immigration on the number of firms, the number of jobs and the entry rate. The increase in formal employment and the reduction in informal employment are confirmed. The only difference is that the effect on wages is negative but smaller and insignificant in the firm or the worker data. As discussed in the introduction, this exercise shows that our results are not driven by a specific set of compliers, nor by specific pairs of sending and receiving regions.

3.5 Changes in the Composition of Firms

In Appendix Section D we investigate different effects on firm composition. Table D.7 shows that there is a shift of firms, and to a less extent jobs, towards services, retail and construction industries, and away from manufacturing. This is intuitive, as retail and services are more labor intensive and require lower and less specific skills than manufacturing, while construction is also a sector that traditionally employs many migrants. Additionally, non-tradables are also more likely to benefit from the positive demand caused by the arrival of immigrants. These new entrants are quite small in size, as Table D.8 shows that there is an increase in the share of firms with up to 5 employees.

Given these compositional effects, and since migration reduces wages and increases

formal employment, one might expect the results to be driven by the entry or survival of low-productivity firms. To test this, we classify firms in four productivity quartiles and estimate the effects of migration on the composition of firms. We do not directly observe firm productivity in the data, so we have to rely on a proxy: average wage paid by the firm (over all workers and years the firm operates). We take into account inflation and spatial differences in costs of living by regressing firms' wage on year and micro-region fixed effects, and use the residual as our measure of productivity. One potential drawback of using wages as proxy for productivity is that since migration reduces wages, one may expect to find mechanically more firms in lower quartiles.

Interestingly, however, the results presented in Appendix Table D.9 suggest that firms that expand in response to a migrant labor supply are in the middle of the productivity distribution rather than the very bottom. We also observe relative decline of top-productivity firms. The effects on the composition of jobs mirror those of the composition of firms. These results cannot be driven by a mechanical composition effect that would see more low-wage firms in municipalities whose wage decrease due to immigration. Instead, our results strongly suggest that immigration favors the entry of firms in the middle of the productivity distribution, which strive in the long-run, and make the formal sector grow. In the next sections, we will present a model and quantitative analysis to explain and rationalize these effects.

3.6 Heterogeneous Effects by Enforcement and Development Levels

In this Section we exploit the enormous disparity across regions in Brazil to investigate two important sources of heterogeneous effects. First, we use the enforcement capacity measure used by Ponczek and Ulyssea (2022) to split the sample between municipalities with weaker and stronger capacity to enforce labor regulations on formal firms (thus directly connected to the intensive margin of informality). The measure enforcement capacity is based on the work of (**?**). It uses the fact labour regulation is enforced by the Ministry of labour using a simple technology: labour inspectors are assigned to labour offices (L.O.) located in municipalities across the country and they travel by car to inspect firms. Hence, greater distances to L.O. imply that firms are less likely to be inspected and enforcement is more likely to be weak (all things equal). We follow Ponczek and Ulyssea (2022) and use distance to the nearest labour

office as a proxy for enforcement capacity in a given local market, conditioning on those labor offices created before our period of analysis. Tables E.1 and E.2 show that the labor market and firm dynamics results are driven by cities with higher enforcement capacity (and potentially stricter enforcement), even after conditioning for different potential confounders. If one thinks of informality as an expression of broader frictions and obstacles to firm growth, then these results imply that the larger the frictions the lower the dividends to urban economies from a growing labor force.

Finally, we examine heterogeneous effects across terciles of GDP per capita at baseline (in 2000). There are huge differences in economic development across regions at baseline: average GDP p.c. in the top tercile is more than twice as large the average in the middle tercile and 4.5 times larger than the bottom one. We show that the effects on workers and firms are concentrated in municipalities in the bottom and middle terciles, which has two important implications. First, the heterogeneity by enforcement levels discussed above is not simply capturing variation in economic development. Second and more broadly, it indicates that urban economies in the lower end of the development spectrum are the ones that benefit the most from the inflows of migrants.

4 Model

This section develops an equilibrium model of firms dynamics and informality. It builds on Hopenhayn (1992) to add key innovations in the entry structure, firms' productivity processes and, crucially, the two margins of informality considered in Ulyssea (2018): (i) whether to register the business or not, the extensive margin; and (ii) whether firms that are formally registered hire their workers with or without a formal contract, the intensive margin. We use this model to quantify the equilibrium effects of an inflow of migrants at the average destination economy, and to gain a deeper understanding of the mechanisms behind our empirical results. We do not attempt to characterize the full spatial equilibrium effects of internal migration flows.

In the model, the informal sector is comprised by informal firms (the extensive margin), while informal workers can be found in both sectors (because of the intensive margin). Every period, there is a continuum of firms that are indexed by their idiosyncratic productivity, θ. All firms have the same technology and use labor as their only input: $f(\ell) = \theta q(\ell)$, where $q(\cdot)$ is increasing and concave. Formal and in-

formal firms operate in the same industry, produce a homogeneous good and face the same prices in a competitive market. For simplicity, we assume that labour is homogeneous.[11] The assumptions regarding market structure imply complete integration between the formal and informal sectors, and are consistent with the evidence that formal and informal firms operate in the same (narrowly defined) industries (Ulyssea, 2020).

Informal firms are able to avoid taxes, but face an "informality cost" captured by $1 \leq \tau_i(\cdot) < \infty$, where $\tau_i', \tau_i'' > 0$. This function is a general formulation for the different costs associated to informality, such as the probability of being inspected by the government, which are likely to be increasing in firms' size (e.g. larger firms are more visible to the government and inspected with higher probability). Informal firm's profit function is given by

$$\Pi_i(\theta, w) = \max_{\ell} \{\theta q(\ell) - \tau_i(\ell) w\} \tag{4}$$

Formal firms must pay revenue and payroll taxes, but can evade the latter by hiring informal workers. However, formal firms face a cost of doing so, which can be rationalized along similar lines as above – e.g. a large formal firm is more likely to be audited by the government. We assume that formal firms' cost of hiring informal workers is given by $1 \leq \tau_f(\ell_i) < \infty$, where also $\tau_f', \tau_f'' > 0$. Hence, formal and informal workers have different marginal costs due to regulations that are imperfectly enforced.[12] As both types of workers are perfect substitutes, at the margin firms hire the cheaper one and there is a unique threshold, $\tilde{\ell}$, above which all additional workers are hired formally. Formal firms' profit function can thus be written as follows:

$$\Pi_f(\theta, w) = \max_{\ell} \{(1 - \tau_y)\theta q(\ell) - C(\ell)\} \tag{5}$$

[11] This assumption can be relaxed both in terms of allowing for different skill levels and for imperfect substitution between migrants and non-migrants. Ulyssea (2018) shows that the skill homogeneity assumption is not consequential to his main quantitative results. Given that we focus on internal migration and our empirical results show that immigration was neutral in terms of skill composition at destination, we believe that the worker homogeneity assumption is a reasonable simplification in our context.

[12] The marginal cost of hiring informal workers is increasing, while the marginal cost of hiring formal workers is constant. The cost for formal firms of hiring informal workers is given by $\tau_f(\ell)w$, while the cost of hiring formal workers is $(1 + \tau_w) w\ell$, where τ_w is the payroll tax.

where τ_y denotes the revenue tax and

$$C(\ell) = \begin{cases} \tau_f(\ell)\, w, & \text{for } \ell \leq \tilde{\ell} \\ \tau_f\left(\tilde{\ell}\right) w + (1+\tau_w)\left(\ell - \tilde{\ell}\right) w, & \text{for } \ell > \tilde{\ell} \end{cases} \tag{6}$$

Additionally, every period firms must pay a fixed cost of operation and the net profit is given by: $\pi_s(\theta, w) = \Pi_s(\theta, w) - \bar{c}_s,\ s = i, f$.

4.1 Entry and Dynamics

Every period, there is a large mass of potential entrants that decide whether to enter the informal or formal sector or not to enter at all. Before entry, they observe their potential long-run (fixed) productivity parameter, ν, which is drawn from the CDF $H(\nu)$. These are assumed to be i.i.d. and entry in one period does not affect the composition of entrants in the following. Entrants in both sectors must pay a fixed entry cost, c_s^e. These parameters will be estimated, but we expect that $c_f^e > c_i^e$ due to regulatory costs associated to creating a formal firm. Informal sector's entry cost can be interpreted as the initial investment necessary to start operating in a given industry.

Post-entry dynamics are driven by the evolution of firms' idiosyncratic productivity, θ, which is the only source of uncertainty to the firm (there are not aggregate shocks). After entry in either sector occurs, the productivity process is characterized by the following expressions:

$$\ln \theta_{j,1} = \ln \nu_j + \ln \epsilon_{j,1} \tag{7}$$

$$\ln \theta_{j,t} = \rho_s \ln \theta_{j,t-1} + (1-\rho_s) \ln \nu_j + \ln \epsilon_{j,t},\ t \geq 2 \tag{8}$$

where j indexes firms, the unexpected shock is i.i.d. and $\ln \epsilon \sim \ln \mathcal{N}(0, \sigma_s^2),\ s = i, f$.

We assume that ν follows a Pareto distribution, which implies that firms' first productivity draw, $\theta_1 = \nu \epsilon_1$, has a Pareto-Lognormal distribution. This distribution is very well-suited to characterize firm size distributions in developing countries (e.g. Ulyssea, 2018). Crucially, Equations 7 and 8 imply that firms' current productivity can differ from their long run productivity level, and there is heterogeneity across firms in terms of life-cycle growth profiles.

We assume that formal firms cannot become informal,[13] and therefore they face a simple stopping-time problem (to remain active or exit). Informal incumbents have an additional option, as they can transit to the formal sector. To formalize, an active informal firm must pay the difference between sectors' entry costs, $\tilde{c}^e = c_f^e - c_i^e$, and after that it will face the regulatory costs implied by formality. In addition to endogenous exit, we assume that both formal and informal firms face an exogenous death shock, denoted by δ_s, $s = i, f$.

The value functions of formal and informal *incumbents*, respectively, can be written in recursive form as follows:

$$\begin{aligned} V_f(\theta, w) &= \pi_f(\theta, w) + (1-\delta_f)\beta \max\left\{0, E_\nu\left[V_f(\theta', w)\left|\theta\right.\right]\right\} && (9) \\ V_i(\theta, w) &= \pi_i(\theta, w) && (10) \\ &+ \beta \max\left\{0, (1-\delta_i) E_\nu\left[V_i(\theta', w)\left|\theta\right.\right], (1-\delta_f) E_\nu\left[V_f(\theta', w)\left|\theta\right.\right] - \tilde{c}^e\right\} \end{aligned}$$

where β denotes the discount factor and the subindex ν in the continuation value indicates that the expectation depends on the firm-specific long run productivity parameter, as described in expressions 7 and 8.

The exit decision is made before future productivity is revealed and it follows a cut-off rule. If $\theta < \underline{\theta}_s$ firms choose to exit, where $\underline{\theta}_s$ is given by

$$E_\nu\left[V_s(\theta', w)\left|\underline{\theta}_s\right.\right] = 0, \;\; s = i, f \tag{11}$$

Entry is also characterized by a threshold rule, as follows:

$$\begin{aligned} E_\nu\left[V_i(\theta, w)\left|\nu = \underline{\nu}_i\right.\right] &= c_i^e && (12) \\ E_\nu\left[V_f(\theta, w) - V_i(\theta, w)\left|\nu = \underline{\nu}_f\right.\right] &= c_f^e - c_i^e && (13) \end{aligned}$$

where $\underline{\nu}_s$ characterizes the last firm to enter sector $s = i, f$.

Finally, informal firms formalize if $\theta \geq \overline{\theta}_i$, where $\overline{\theta}_i$ is given by:

$$E_\nu\left[V_f(\theta', w) - V_i(\theta', w)\left|\overline{\theta}_i\right.\right] = \tilde{c}^e \tag{14}$$

[13] Once the firm is visible to the government, it is very hard to become informal and therefore "invisible" again, as the government has formal firms' complete information. It is still the case that firms could change locations to become informal, which in our model would correspond to exiting the formal sector and rejoining the pool of potential entrants (but with no memory of previous experiences).

The *timing* of informal firms' decisions is the following. At the beginning of the period, firms draw their productivity shock, θ, and decide how much to produce (i.e. how much labor to hire). If $\theta \in \left[\underline{\theta}_i, \overline{\theta}_i\right)$, they will start next period in the informal sector again; if $\theta \geq \overline{\theta}_i$, they will pay $\tilde{c}^e$ and start next period in the formal sector; and if $\theta < \underline{\theta}_i$, they exit. The timing for formal firms is the same, except that they only choose whether to stay or exit.

4.2 Steady state equilibrium

To close the model, we assume that there is a representative household that inelastically supplies $\overline{L}$ units of labor and that derives utility from consuming the final good. A labor supply shock in our setting therefore corresponds to a once-and-for-all increase in $\overline{L}$. Households consume all of their income, which is given by $w\overline{L} + \Pi + T$, where Π denotes the mass of profits in the economy, $w\overline{L}$ is total wage income and T denotes total tax revenues. The model therefore also incorporates the positive demand shock induced by a migration shock.

We focus on stationary competitive equilibria, which correspond to a set of allocations, wage, cutoffs and measures of firms such that they remain constant over time and the following conditions hold in every period: (i) product and labor markets clear;[14] (ii) the cutoffs $\left(\underline{\theta}_s, \overline{\theta}_i, \underline{\nu}_s\right)$, $s = i, f$, are determined according to (11), (14), (12) and (13), respectively; (iii) entry conditions (12) and (13) hold. In the Appendix F we show that there exists a unique stationary competitive equilibrium where both the formal and informal sector exist, and both sectors have positive entry and exit (Proposition 1).

Appendix F provides other characterization results that shed further light on the model's main mechanisms. In particular, we show that the productivity distribution among active firms within a sector and cohort is increasing in the cohort's age (Proposition 2). Put differently, the productivity distribution of older cohorts first order stochastically dominates that of younger cohorts. From this result, it follows that any function that is increasing in firms' productivity, θ, will also be increasing in cohort's age. In particular, this is true for the survival rate, firms' average size and average revenue in *both the formal and informal sectors* (Corollary 1). These results also imply that the average share of informal workers within formal firms (intensive

[14] We normalize the price of the final good to one throughout the analysis.

margin) and the average share of informal firms (extensive margin) within a given cohort decline with the cohort's age (Corollaries 2 and 3, respectively). Importantly, these results constitute falsifiable predictions and show that the model has empirical content.

4.3 Discussion

A recent but growing literature has documented the decline in startup rates, and its adverse effects on job creation, labor reallocation, firms and employment growth in the U.S. (e.g. Decker et al., 2014; Pugsley and Sahin, 2019). This literature points at the slowdown in labor supply growth as the main determinant of this "startup deficit" (Hopenhayn et al., 2018; Karahan et al., 2019). The effects of internal migration on receiving local economies represent the mirror image of the phenomena emphasized in this literature, as some receiving regions experience large labor supply shocks. An increase in labor supply must be accompanied by an increase in labor demand, which by its turn requires a decline in real wages that allows both incumbents' labor demand to expand and new firms to enter. Incumbents are limited by scale, so ultimately this increase in the labor supply must be met by greater firm entry, especially in face of a substantial change in labor supply. In equilibrium, not only entry rates are affected, but also exit rates, and firm size distribution (Hopenhayn et al., 2018).

These effects on firm dynamics are likely to have first order effects on labor markets. It is less clear what the net effects will be in terms of overall firm and labor informality, and aggregate outcomes such as output per worker. If all migrants are absorbed by informal jobs in informal firms and low-productivity formal firms, the net result will most likely be an increase in informality among firms and workers. However, if the decline in wages induces greater entry also of firms with higher growth potential – either in the informal or formal sectors – then this can lead to more firm growth in the formal sector, and to a new steady state equilibrium with higher numbers of formal firms and formal jobs. The results discussed in Section 3 suggest that the latter force dominates, although these are relative effects. To get at equilibrium effects, we move to the model and estimation and counterfactual analysis in Section 5.

5 Quantitative Analysis

5.1 Model Estimation

In order to estimate the model discussed in Section 4 and use it to perform counterfactual simulations, it is necessary to first parameterize the remaining objects in the model's structure that were left unspecified. Starting by the production function, we assume a simple span-of-control formulation: $y(\theta, \ell) = \theta\ell^{\alpha}$, $0 < \alpha < 1$. The cost functions are defined as in Ulyssea (2018): $\tau_s(\ell) = \left(1 + \frac{\ell}{\varphi_s}\right)\ell$, where $\varphi_s > 0$, $s = i, f$. These cost functions are increasing and convex in firm size, and have a very simple formulation that depends on a single parameter, φ_s. The larger the φ_s, the more informal firms can grow before it becomes too costly to be informal, or the more informal workers formal firms are able to hire before it becomes too costly.

To estimate the model, we use a two-step Simulated Method of Moments (SMM) procedure. In the first step, we use the RAIS establishment-level panel to estimate the persistence parameter of formal firms' productivity process, which gives $\rho_f = 0.921$ (see Appendix G for details). We set the tax parameters to their statutory values: $\tau_w = 0.375$ and $\tau_y = 0.293$. The value of τ_w corresponds to the main regulatory costs that are proportional to firms' payroll (social security contribution, direct payroll tax and severance contributions). The τ_y corresponds to the federal VAT taxes (IPI and PIS/COFINS), and excludes state-level value-added taxes. The latter vary a lot across state borders and the system is characterized by an intricate system of tax substitution along the production chain. As in Ulyssea (2018), the Pareto distribution scale parameter (ν_0) is set so that the firms' minimum size is one employee, while formal sector's fixed cost of operation parameter is set to be 70 percent of the baseline wage. Table 5 describes all parameter values.

The second step of the estimation procedure takes the first-step parameters as given to estimate the remaining 12 parameters of the model, which are the following: $\Omega = \left\{\varphi_f, \varphi_i, \delta_i, \delta_f, \bar{c}_i, \xi, c_f^e, c_i^e, \alpha, \sigma_i, \sigma_f, \rho_i\right\}$, where φ_i and φ_f are the parameters of extensive and intensive margins' cost functions, respectively; δ_i and δ_f are the exogenous death shocks in the informal and formal sectors, respectively; $\bar{c}_i$ determines the per-period fixed cost of operation; ξ is the Pareto shape parameter; c_f^e and c_i^e are the formal and informal entry costs, respectively; α is the span-of-control; σ_i^2 and σ_f^2 are the informal and formal variances of the productivity shock; and ρ_i is the persistence

of the productivity process in the informal sector. The vector Ω is estimated by minimizing the distance between the vector of data moments, $\hat{m}_N$, and the same vector of moments computed from simulated data generated by the structural model. In Appendix Section G we discuss the implementation details of the SMM estimator.

Moments and estimation results

We use the data sources described in Section 2 to estimate the data moments used in the SMM estimation. Since the ECINF survey (*Pesquisa de Economia Informal Urbana*) is only available in 2003, for the model estimation we also restrict the RAIS (*Relação Anual de Informações Sociais*) to the same year. We use both data sets to compute the following moments: overall share of informal firms and by size brackets, ≤ 2 and $[3, 5]$ employees (source: ECINF and RAIS); average share of informal workers within formal firms (source: ECINF); formal firm growth at ages 5 and 10, relative to age 1 (source: RAIS); informal firm growth at age 5 relative to age 1 (source: ECINF); formal firms size distribution by size brackets, ≤ 5, $(5, 10]$, $(11, 20]$, $(20, 50]$,> 50 employees (source: RAIS); informal firms size distribution by size brackets, ≤ 2, $(2, 5]$ employees (source: ECINF). Because the Demographic Census is only available every ten years, we use the 2003 National Household Survey (PNAD) to compute the share of informal workers in the economy as another moment to be matched in the estimation procedure.

Table 5 shows the estimation results. The estimated formal sector's entry cost is more than twice as large as informal sector's. The values are denominated in *Reais* of 2003 and are not negligible: formal sector's entry costs correspond to more than 30 times the monthly national minimum wage at the time. Informal firms face a relatively high exogenous death shock of nearly 15 percent ($\delta_i = 0.148$), which is more than twice as large as in the formal sector. Apart from the exogenous exit shock, informal firms do not face higher uncertainty, as the variances of the productivity innovation in both sectors, σ_s^2, are statistically equal. The same is true for persistence of the productivity process in both sectors.

Model fit and identification

Table 6 shows how the model fits the share of informal workers and firms, as well as the targeted moments relative to firm size distribution. The model fits these

moments quite well, which is not surprising since these are targeted in the estimation procedure. To further assess how the model fits the data, we examine non-targeted moments in Figure 4. Panels (a) and (b) show that the model reproduces well the behavior observed for the extensive and intensive margins of informality, respectively. Even though it predicts a faster decline in the share of informal firms than what is observed in the data, the fit for the intensive margin is very good and the model is able to match well the behavior of the average share of informal workers within formal firms relative to firm size.

We also use the panel structure of RAIS to examine how the model fits moments directly related to firms' dynamics. As panel (c) shows, the model fits well the growth profile for formal firms up to age 11. The simulated data shows a more concave profile, however, which implies that the model slightly overestimates the accumulated growth for ages 3 to 9, but eventually both model and data converge at age 11. Similarly, panel (d) in Figure 4 shows that the model reproduces very well the behavior of the autocorrelations of log-employment relative to log-employment at age one.

It is typically the case in such models, it is not possible to provide a formal identification argument. In the Appendix G, however, we provide a careful discussion of which dimensions of the data provide the variation that allows us to estimate the model's parameters.[15] We also follow Adda et al. (2017) and investigate how informative the moments used in the estimation are.

5.2 Counterfactuals

In this section, we use the estimated model to asses the equilibrium effects of a substantial in-migration shock. We simulate a once-and-for-all increase in the aggregate labor supply of 10%, which is around the 75th percentile of the distribution of immigration rates we observe in our data. We simulate this shock in two different environments. The first is the baseline Brazilian economy in 2003, which is described by the model and parameters estimates discussed above. This is arguably a low enforcement and high informality environment. Second, we simulate a new baseline economy which is the same as the Brazilian economy in 2003, but with a 50% increase in the intensity of enforcement on both the intensive and extensive margins of informality. This is a large but plausible difference, given the variation in enforcement

[15]This discussion is thus closer to the concept of *sensitivity* proposed by Andrews et al. (2017).

levels across regions in Brazil (Ponczek and Ulyssea, 2022).

We start by examining the effects of the supply shock on firm growth. Figure 5 shows that there is a modest effect on firm growth rates under both scenarios, but which is larger at older ages and more pronounced in the high enforcement scenario. Figure 6 shows the changes in the allocation of formal firms and workers across the quartiles of firm productivity distribution. This is the analog to the empirical exercise discussed in section 3, but using firms' model-equivalent to TFPQ (instead of wages as a proxy). The model-based results are very consistent with the IV results: employment grows more in firms in the middle of the distribution at the top. The effects on the number of firms is very small and not reported.

These reallocation effects suggest that the labor supply shock could have positive, but perhaps limited aggregate effects on TFP and output. Table 7 shows the different aggregate effects. The first two columns show the baseline economy and the new steady state after the 10% labor supply shock hits (columns "Pre" and "Post", respectively). Interestingly, the results largely confirm our empirical analysis: there is a small reduction in the share of informal workers and a sizable increase in the number of formal firms. The model also shows that this is followed by a small decline in the share of informal firms in the economy. Wages fall, as expected, but the elasticity is far from one, which indicates that the increase in labor supply is being partially met by an increase in production and therefore in the demand for labor. The size distribution shifts slight away from very small firms, but even then average productivity remains basically constant. Both output and income (welfare) increase, but by less than the increase in labor supply, which implies that output and income per capita are falling.

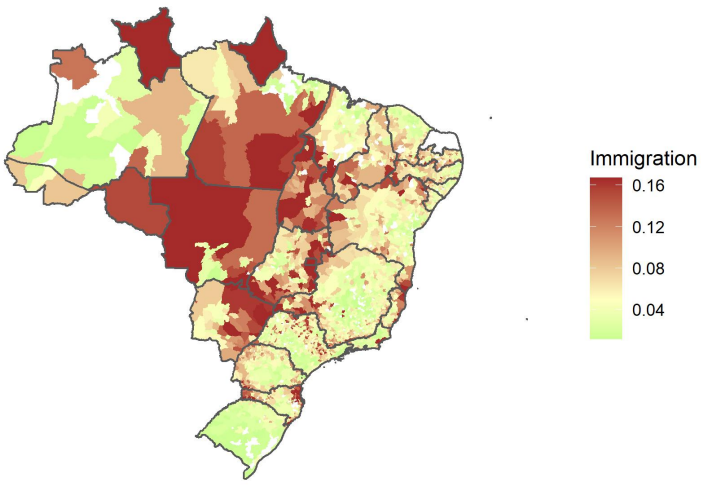
When we move to the analysis of the same shock but in a high enforcement economy/region, the results on informality and mass of formal firms get amplified. This is intuitive, as in a high enforcement economy there is less room for expansion in the informal sector and therefore formal firms can take greater advantage of this supply shock. This translates into higher effects on total output and income, which are still not enough to compensate the increase in total population. Importantly, however, tax revenues increase substantially. Hence, there might be scope for governments to use these additional resources to alleviate potentially negative consequences from migration.

6 Conclusion

In this paper, we study the effect of internal migration on firms dynamics, using data on all formal firms and all workers in Brazil between 1995 and 2018. To identify the causal effect of immigration on firms and workers, we use a shift-share instrument, which relies on agricultural price shocks that push rural workers to urban areas. We find that immigration fosters formal employment among resident workers and lowers their wages. We show that these results are driven by firm dynamics, with increased firm entry in the formal sector and greater churn. We develop a structural model of firm dynamics which we estimate based on the same data and obtain similar results: large labor supply shocks can generate substantial gains in formal employment and formal firm growth. These results run counter to the common narrative that rural-urban migration only increases the number of informal or under-employed workers in developing country cities, a view that dates back to (Harris and Todaro, 1970). Instead, it suggests that developing countries might experience demographic dividends from internal migration, with active firm creation. The recent literature highlights the opposite phenomenon in the US, with declining demographics leading to a "start-up deficit" (Pugsley and Sahin, 2019; Karahan et al., 2019).

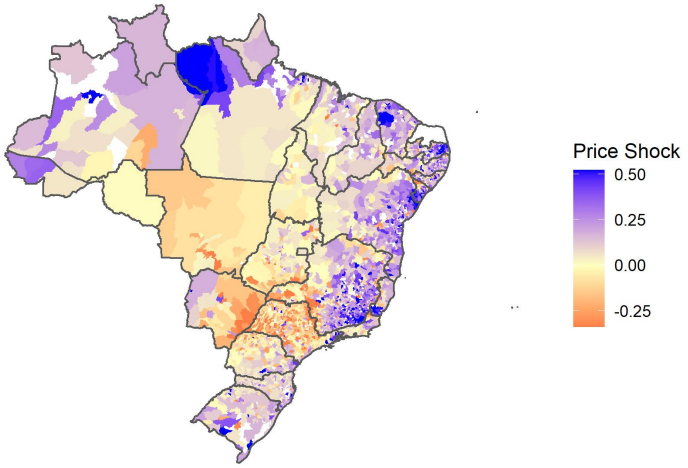
Tables and Figures

Figure 1: In-Migration, 2000-2010



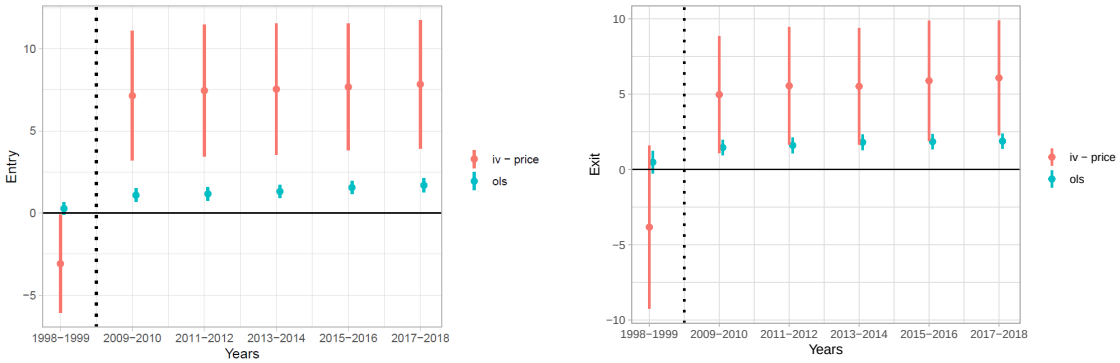
Notes: Computed using the Decennial Population Census. Darker areas denote higher in-migration rates.

Figure 2: Push Shocks: Crop Prices



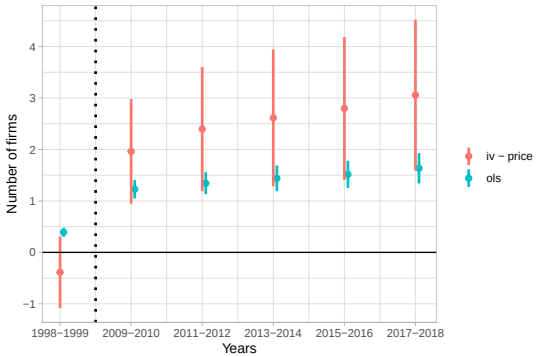
Notes: The Price shocks are constructed according to expression 1 in the text.

Figure 3: Dynamic Effects

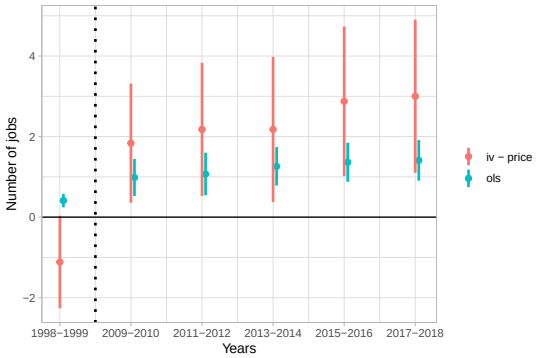


(a) Entry

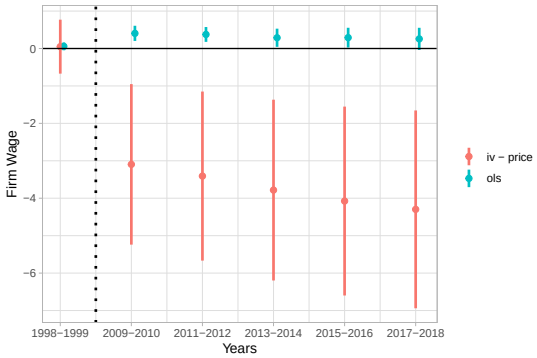
(b) Exit



(c) Number of firms



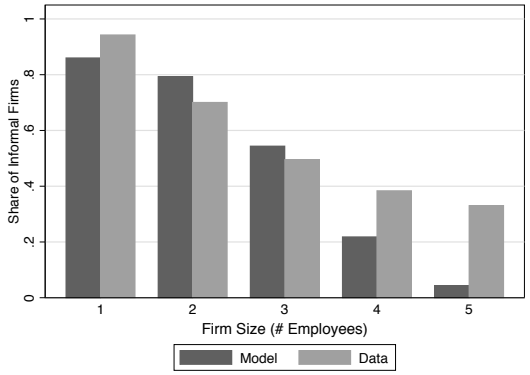
(d) Number of jobs



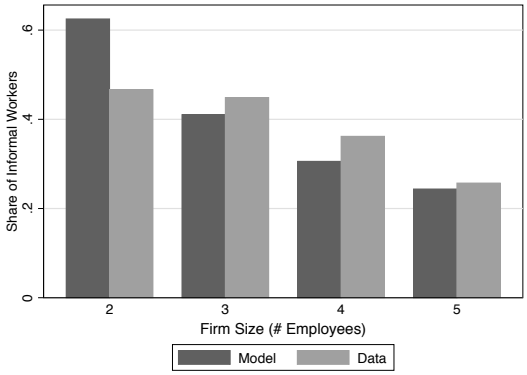
(e) Firm wage

Notes: The Price shocks are constructed according to expression 1 in the text.

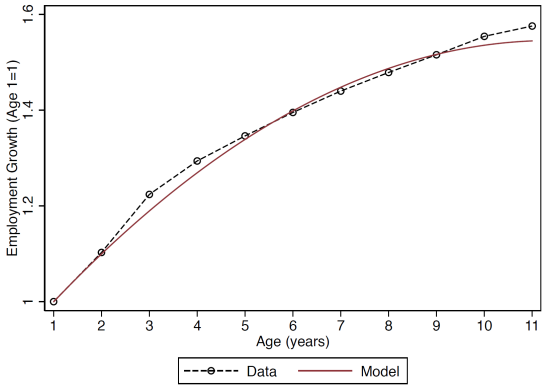
Figure 4: Model fit: non-targeted moments



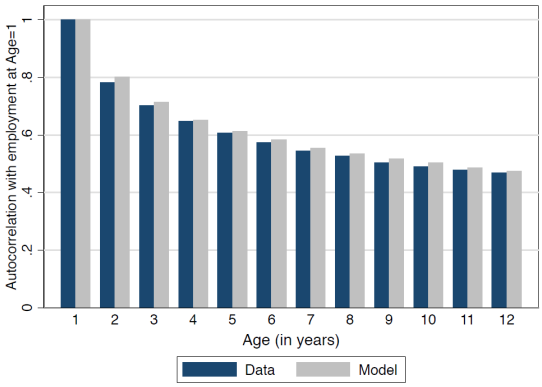
(a) Extensive Mg.



(b) Intensive Mg.



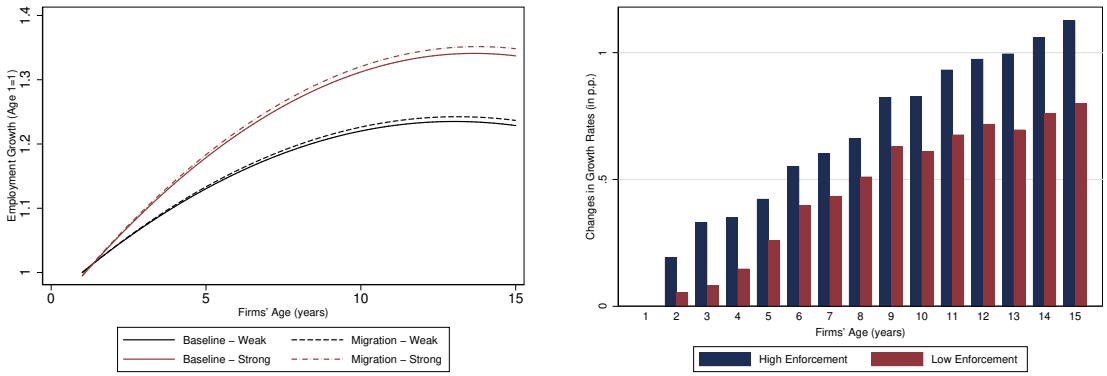
(c) Formal Firms' growth



(d) Log-Employment Autocorrelations

Notes: Size is measured as number of employees. For panels (c)–(e), both simulated and empirical moments use only formal firms and their formal workers. The empirical moments are computed using the panel data from RAIS.

Figure 5: Changes in firm growth



(a) Growth profiles

(b) Changes in growth rates

Figure 6: Changes in the share of formal workers across quartiles of firm productivity

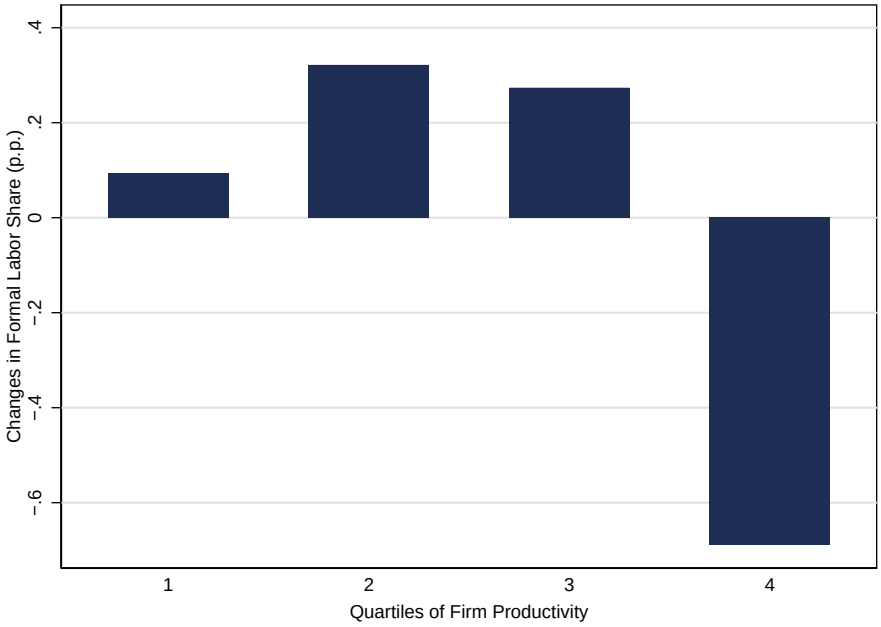


Table 1: Descriptive Statistics

	2000		2010	
	Mean	SD	Mean	SD
Panel A: Socio-Demographics				
% Female	0.483	0.014	0.482	0.013
% Young	0.128	0.021	0.102	0.016
% Low-skilled	0.72	0.096	0.579	0.094
Panel B: Labor Market Outcomes				
% Overall wage employment	0.332	0.061	0.4	0.071
% Formal wage employment	0.229	0.076	0.309	0.096
% Informal wage employment	0.103	0.036	0.09	0.037
Log overall monthly wage	6.886	0.415	6.992	0.321
Log formal monthly wage	7.017	0.361	7.074	0.28
Log informal monthly wage	6.578	0.391	6.712	0.296
Panel C: Firm Outcomes				
Log number of firms	8.728	2.342	9.285	2.196
Log entry	7.168	2.274	7.58	2.117
Log exit	6.917	2.347	7.305	2.141
Log number of jobs	11.303	2.514	11.954	2.389
Log firm monthly wage	6.398	0.457	6.683	0.321
Panel D: Immigration				
Immigration rate			0.176	0.092
Immigration rate (State to State)			0.07	0.054
Panel E: Population				
Population	25117	155203	31413	183006

Notes: For all variables there are 3548 observations in both 2000 and 2010. Mean and SD in panels A-D are weighted by population in 2000.

Table 2: Effects of Immigration on Workers (2010)

	Wage employment			Log monthly wage		
	Overall (1)	Formal (2)	Informal (3)	Overall (4)	Formal (5)	Informal (6)
Panel A: OLS						
Immigration	0.037	0.105	−0.068	0.062	0.031	0.034
	(0.019)	(0.023)	(0.014)	(0.076)	(0.068)	(0.092)
Panel B: IV-Price						
Immigration	0.102	0.397	−0.294	−1.575	−2.149	−1.864
	(0.101)	(0.147)	(0.099)	(0.568)	(0.667)	(0.726)
F Statistic (IV)	16.87	16.87	16.87	16.87	16.87	16.87
Baseline Mean	0.332	0.229	0.103	–	–	–
Observations	3548	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions are controlled for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population at the destination municipality in the previous census.

Table 3: Effects of Immigration on Firms (2011-2012)

	Nb firms (1)	Entry rate (2)	Exit rate (3)	Nb jobs (4)	Firm wage (5)
Panel A: OLS					
Immigration	1.344	1.167	1.596	1.071	0.375
	(0.109)	(0.202)	(0.273)	(0.269)	(0.101)
Panel B: IV - Price					
Immigration	2.395	7.457	5.548	2.178	−3.408
	(0.615)	(2.040)	(1.999)	(0.843)	(1.152)
F Statistic (IV)	16.87	16.87	16.87	16.87	16.87
Observations	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school in the destination municipality in the previous census. All regressions are weighted by the population pin the destination municipality in the previous census.

Table 4: Effects of Immigration on Firms (2011-2012) - Enforcement Heterogeneity

	Nb firms (1)	Entry rate (2)	Exit rate (3)	Nb jobs (4)	Firm wage (5)
IV - Price					
Immigration	0.823	0.452	1.484	1.083	0.437
	(0.618)	(1.349)	(1.292)	(0.768)	(0.792)
Immigration*High Enforcement	2.167	9.452	5.297	1.403	−5.494
	(0.957)	(3.432)	(3.410)	(1.188)	(2.049)
High Enforcement	−0.154	−0.736	−0.476	−0.135	0.325
	(0.068)	(0.254)	(0.266)	(0.085)	(0.146)
Observations	3547	3547	3547	3547	3547

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school in the destination municipality in the previous census. All regressions are weighted by the population pin the destination municipality in the previous census. High Enforcement – a dummy variable for the MCAs with a minimum distance to the labor office from the MCA's centroid over the median.

Table 5: Estimation Results

Parameter	Description	Source	Value	SE
First Step				
τ_w	Payroll Tax	Statutory values	0.375	–
τ_y	Revenue Tax	Statutory values	0.293	–
ρ	Productivity Process: Persistence Parameter	GMM Estimation	0.92	–
ν_0	Pareto's Location Parameter	Calibrated	7.3	–
γ_f	Per-period fixed cost of operation (Formal)	Calibrated	0.7	–
Second Step				
φ_f	Intensive margin: $\tau_f = \left(1 + \frac{\ell}{\varphi_f}\right)\ell$	SMM Estimation	6.450	0.228
φ_i	Extensive margin: $\tau_i = \left(1 + \frac{\ell}{\varphi_i}\right)\ell$	SMM Estimation	5.427	0.303
δ_i	Informal death shock	SMM Estimation	0.148	0.015
δ_f	Formal death shock	SMM Estimation	0.066	0.011
γ_i	Informal, per-period fixed cost of operation	SMM Estimation	0.350	0.161
ξ	Pareto shape parameter	SMM Estimation	3.801	0.092
$c_f^{e\dagger}$	Formal sector's entry cost	SMM Estimation	7,400	3,383
$c_i^{e\dagger}$	Informal sector's entry cost	SMM Estimation	2,800	598
α	Span-of-control	SMM Estimation	0.643	0.218
σ_i	Informal productivity process: SD	SMM Estimation	0.144	0.053
σ_f	Formal productivity process: SD	SMM Estimation	0.148	0.032
ρ_i	Informal productivity process: persistence	SMM Estimation	0.935	0.091

† Estimates and SD expressed in R$ of 2003.

Table 6: Model Fit: targeted moments

	Model	Data
Share Informal workers	0.305	0.298
Share Informal Firms	0.704	0.696
Informal Firms Size Distribution		
≤ 2 employees	0.929	0.957
≤ 5 employees	1.000	0.998
Formal Firms Size Distribution		
≤ 5 employees	0.694	0.697
6 to 10	0.134	0.144
11 to 20	0.092	0.083
21 to 50	0.056	0.048
> 50	0.024	0.028

Notes: Data moments computed using the RAIS, ECINF and PNAD data sets.

Table 7: Aggregate Effects: Low vs. High Enforcement

	Low Enforcement		High Enforcement	
	Pre	Post	Pre	Post
Share Informal Workers	0.305	0.290	0.175	0.166
Share Informal Firms	0.704	0.694	0.582	0.545
% Formal firms with				
1 to 5 employees	0.694	0.675	0.696	0.700
6 to 10 employees	0.134	0.141	0.147	0.144
11 to 20 employees	0.092	0.098	0.089	0.087
21 to 50 employees	0.056	0.060	0.048	0.048
> 50 employees	0.024	0.026	0.020	0.020
Wages	1.000	0.972	1.000	0.970
Number of firms	1.000	1.013	1.000	1.043
Number of formal firms	1.000	1.046	1.000	1.136
Output	1.000	1.065	1.000	1.078
Taxes	1.000	1.081	1.000	1.090
Total Income	1.000	1.052	1.000	1.054

References

Adão, R., M. Kolesár, and E. Morales (2019). Shift-share designs: Theory and inference. *The Quarterly Journal of Economics 134*(4), 1949–2010.

Adda, J., C. Dustmann, and K. Stevens (2017). The career costs of children. *Journal of Political Economy 125*(2), 293–337.

Akcigit, U., S. Baslandze, and S. Stantcheva (2016). Taxation and the international mobility of inventors. *American Economic Review 106*(10), 2930–81.

Albert, C., P. Bustos, and J. Ponticelli (2021, June). The Effects of Climate Change on Labor and Capital Reallocation. CEPR Discussion Papers 16312, C.E.P.R. Discussion Papers.

Alfonsi, L., O. Bandiera, V. Bassi, R. Burgess, I. Rasul, M. Sulaiman, and A. Vitali (2020). Tackling youth unemployment: Evidence from a labor market experiment in uganda. *Econometrica 88*(6), 2369–2414.

Altonji, J. G., A. A. Smith, and I. Vidangos (2013). Modeling earnings dynamics. *Econometrica 81*(4), 1395–1454.

Andrews, I., M. Gentzkow, and J. M. Shapiro (2017). Measuring the sensitivity of parameter estimates to estimation moments. *The Quarterly Journal of Economics 132*(4), 1553–1592.

Arellano, M. and S. Bond (1991). Some tests of specification for panel data: Monte carlo evidence and an application to employment equations. *The Review of Economic Studies 58*(2), pp. 277–297.

Bandiera, O., V. Bassi, R. Burgess, I. Rasul, M. Sulaiman, and A. Vitali (2021). The search for good jobs: evidence from a six-year field experiment in uganda. *Available at SSRN 3910330*.

Bertoli, S., F. Docquier, H. Rapoport, and I. Ruyssen (2020). Weather Shocks and Migration Intentions in Western Africa: Insights from a Multilevel Analysis. CESifo Working Paper Series 8064.

Blundell, R. and S. Bond (1998). Initial conditions and moment restrictions in dynamic panel data models. *Journal of Econometrics 87*, 115–143.

Borjas, G. J. (2003, November). The Labor Demand Curve Is Downward Sloping: Reexamining The Impact Of Immigration On The Labor Market. *The Quarterly Journal of Economics 118*(4), 1335–1374.

Borusyak, K., P. Hull, and X. Jaravel (2018). Quasi-experimental shift-share research designs. Technical report, National Bureau of Economic Research.

Bruins, M., J. A. Duffy, M. P. Keane, and A. A. Smith Jr (2015). Generalized indirect inference for discrete choice models. Yale University, mimeo.

Bustos, P., J. M. Castro Vincenzi, J. Monras, and J. Ponticelli (2018, December). Structural Transformation, Industrial Specialization, and Endogenous Growth. CEPR Discussion Papers 13379, C.E.P.R. Discussion Papers.

Card, D. (2001). Immigrant inflows, native outflows, and the local labor market impacts of higher immigration. *Journal of Labor Economics 19*(1), 22–64.

Card, D. and J. DiNardo (2000). Do immigrant inflows lead to native outflows? *American Economic Review 90*(2), 360–367.

Clemens, M. A. and J. Hunt (2019, August). The Labor Market Effects of Refugee Waves: Reconciling Conflicting Results. *ILR Review 72*(4), 818–857.

Corbi, R., T. Ferraz, and R. Narita (2021). Internal migration and labor market adjustments in the presence of nonwage compensation. Manuscript.

de Paula, A. and J. A. Scheinkman (2010). Value-added taxes, chain effects, and informality. *American Economic Journal: Macroeconomics 2*(4), 195–221.

Decker, R., J. Haltiwanger, R. Jarmin, and J. Miranda (2014). The role of entrepreneurship in us job creation and economic dynamism. *Journal of Economic Perspectives 28*(3), 3–24.

Dix-Carneiro, R., P. K. Goldberg, C. Meghir, and G. Ulyssea (2021). Trade and informality in the presence of labor market frictions and regulations. Technical report, National Bureau of Economic Research.

Duffie, D. and K. Singleton (1993). Simulated moments estimation of markov models of asset prices. *Econometrica 61*(4), 929–952.

Dustmann, C. and A. Glitz (2015). How Do Industries and Firms Respond to Changes in Local Labor Supply? *Journal of Labor Economics 33*(3), 711 – 750.

El Badaoui, E., E. Strobl, and F. Walsh (2017). Impact of Internal Migration on Labor Market Outcomes of Native Males in Thailand. *Economic Development and Cultural Change 66*, 147–177.

Engbom, N. and C. Moser (2021). Earnings inequality and the minimum wage: Evidence from brazil. *NBER Working Paper* (No.28831).

Fields, G. S. (1975). Rural-urban migration, urban unemployment and underemployment, and job-search activity in ldcs. *Journal of development economics 2*(2), 165–187.

Gollin, D., D. Lagakos, and M. E. Waugh (2014). The agricultural productivity gap. *The Quarterly Journal of Economics 129*(2), 939–993.

Gourinchas, P.-O. and J. Parker (2002). Consumption over the life cycle. *Econometrica 70*(1), 47–89.

Harris, J. R. and M. P. Todaro (1970). Migration, unemployment and development: A two-sector analysis. *The American Economic Review 60*(1), pp. 126–142.

Hopenhayn, H., J. Neira, and R. Singhania (2018). From population growth to firm demographics: Implications for concentration, entrepreneurship and the labor share. Technical report, National Bureau of Economic Research.

Hopenhayn, H. A. (1992, September). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica 60*(5), 1127–1150.

Hsieh, C.-T. and P. J. Klenow (2014). The life cycle of plants in india and mexico. *The Quarterly Journal of Economics 129*(3), 1035–1084.

Imbert, C., M. Seror, Y. Zhang, and Y. Zylberberg (2022, June). Migrants and firms: Evidence from china. *American Economic Review 112*(6), 1885–1914.

Jaeger, D. A., J. Ruist, and J. Stuhler (2018, February). Shift-Share Instruments and the Impact of Immigration. CEPR Discussion Papers 12701, C.E.P.R. Discussion Papers.

Jedwab, R., L. Christiaensen, and M. Gindelsky (2017). Demography, urbanization and development: Rural push, urban pull and...urban push? *Journal of Urban Economics 98*(C), 6–16.

Karahan, F., B. Pugsley, and A. Şahin (2019). Demographic origins of the startup deficit. Technical report, National Bureau of Economic Research.

Keane, M. and A. A. Smith (2003). Generalized indirect inference for discrete choice models. Yale University, mimeo.

Kerr, S. P., W. R. Kerr, and W. F. Lincoln (2015). Skilled Immigration and the Employment Structures of US Firms. *Journal of Labor Economics 33*(S1), S147 – S186.

Kleemans, M. and J. Magruder (2018, August). Labour Market Responses To Immigration: Evidence From Internal Migration Driven By Weather Shocks. *Economic Journal 128*(613), 2032–2065.

Lewis, E. (2011). Immigration, Skill Mix, and Capital Skill Complementarity. *The Quarterly Journal of Economics 126*(2), 1029–1069.

Mitaritonna, C., G. Orefice, and G. Peri (2017). Immigrants and firms' outcomes: Evidence from france. *European Economic Review 96*, 62–82.

Moser, P., A. Voena, and F. Waldinger (2014). German jewish émigrés and us invention. *American Economic Review 104*(10), 3222–55.

Olney, W. W. (2013). Immigration and firm expansion. *Journal of Regional Science 53*(1), 142–157.

Peri, G. (2012, February). The Effect Of Immigration On Productivity: Evidence From U.S. States. *The Review of Economics and Statistics 94*(1), 348–358.

Peters, M. (2022). Market size and spatial growth: Evidence from germany's postwar population expulsions. Technical report, NBER WP 29329.

Peters, M. and C. Walsh (2022). Population growth and firm-product dynamics. Technical report, Yale.

Ponczek, V. and G. Ulyssea (2022). Enforcement of labour regulation and the labour market effects of trade: Evidence from brazil. *The Economic Journal 132*.

Pugsley, B. and A. Sahin (2019). Grown-up business cycles. *The Review of Financial Studies 32*(3), 1102–1147.

Santangelo, G. (2016). Firms and farms: The impact of agricultural productivity on the local indian economy. Manuscript.

Sterk, V., P. Sedláček, and B. Pugsley (2021). The nature of firm growth. *American Economic Review 111*(2), 547–79.

Tauchen, G. (1986). Finite state markov-chain approximation to univariate and vector autoregressions. *Economic Letters 20*, 177–81.

The World Bank (2012). World development report 2013: Jobs. Technical report.

Ulyssea, G. (2018). Firms, informality, and development: Theory and evidence from Brazil. *American Economic Review 108*(8), 2015–47.

Ulyssea, G. (2020). Informality: Causes and consequences for development. *Annual Review of Economics 12*, 525–546.

UNCTAD (2021). Handbook of statistics. Technical report, United Nations Conference on Trade and Development.

Vicente-Serrano, S. M., S. Beguería, and J. I. López-Moreno (2010). A multiscalar drought index sensitive to global warming: the standardized precipitation evapotranspiration index. *Journal of climate 23*(7), 1696–1718.

Windmeijer, F. (2005). A finite sample correction for the variance of linear efficient two-step gmm estimators. *Journal of Econometrics 126*(1), 25 – 51.

APPENDIX

A First stage of shift-share instruments

Table A.1: First stage

	Immigration	
	(1)	(2)
Price	−0.024	
	(0.003)	
Drought		0.016
		(0.003)
Observations	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

B Additional Results for Workers

Table B.1: Descriptive Statistics by Non-migrants and Migrants

	2000		2010	
	Mean	SD	Mean	SD
Panel B: Labor Market Outcomes for Non-migrants				
% Overall wage employment	0.329	0.059	0.395	0.068
% Formal wage employment	0.229	0.074	0.306	0.094
% Informal wage employment	0.101	0.036	0.089	0.038
Log total monthly wage	6.862	0.433	6.963	0.324
Log formal monthly wage	6.988	0.382	7.042	0.283
Log informal monthly wage	6.553	0.405	6.688	0.3
Panel B: Labor Market Outcomes for Migrants				
% Overall wage employment	0.344	0.076	0.425	0.087
% Formal wage employment	0.231	0.088	0.324	0.109
% Informal wage employment	0.113	0.038	0.101	0.038
Log total monthly wage	6.969	0.376	7.143	0.345
Log formal monthly wage	7.119	0.337	7.238	0.319
Log informal monthly wage	6.654	0.381	6.826	0.325

Notes: Mean and SD in panels A-D are weighted by population in 2000.

Table B.2: Composition effects of workers

	Female (1)	Low Skill (2)	Young (3)	Delta Population (4)	Out-Migration (5)
IV-Price					
Immigration	−0.083	0.029	0.257	0.102	−0.846
	(0.030)	(0.210)	(0.058)	(0.503)	(0.503)
Observations	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions have only one regressor: immigration. All regressions are weighted by the population in the destination municipality in the previous census.

Table B.3: Labor Market Effects for Non-migrants

	Wage employment			Log monthly wage		
	Overall (1)	Formal (2)	Informal (3)	Overall (4)	Formal (5)	Informal (6)
IV-Price						
Immigration	0.096	0.372	−0.276	−2.050	−2.596	−2.252
	(0.101)	(0.144)	(0.099)	(0.678)	(0.781)	(0.818)
Observations	3548	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. High Skilled workers are workers who completed secondary education. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Table B.4: Labor Market Effects by Skill Level

	Wage employment			Log monthly wage		
	Overall (1)	Formal (2)	Informal (3)	Overall (4)	Formal (5)	Informal (6)
Panel A: IV-Price - High-Skilled Workers						
Immigration	0.133	0.372	−0.239	−1.292	−1.598	−1.042
	(0.123)	(0.171)	(0.103)	(0.445)	(0.514)	(0.658)
Observations	3548	3548	3548	3548	3527	3514
Panel B: IV-Price - Low-Skilled Workers						
Immigration	0.044	0.329	−0.284	−1.847	−2.252	−2.407
	(0.105)	(0.108)	(0.094)	(0.730)	(0.777)	(0.970)
Observations	3548	3548	3548	3548	3546	3548

Notes: Robust standard errors in parenthesis. High Skilled workers are workers who completed secondary education. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Table B.5: Labor Market Effects using Hours

	Share of hours			Log hourly wage		
	Overall (1)	Formal (2)	Informal (3)	Overall (4)	Formal (5)	Informal (6)
IV-Price						
Immigration	−0.011	0.396	−0.407	−1.205	−1.850	−1.486
	(0.107)	(0.143)	(0.126)	(0.504)	(0.603)	(0.697)
Observations	3548	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. High Skilled workers are workers who completed secondary education. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Table B.6: Effects of Immigration on Occupational Composition

	Formal (1)	Informal (2)	Non-emp (3)	Self-emp (4)
Immigration	0.397	−0.294	0.082	−0.039
	(0.147)	(0.099)	(0.127)	(0.047)
Baseline Mean	0.229	0.103	0.435	0.119
Observations	3548	3548	3548	3548
	Employer (5)	Domestic (6)	Public (7)	Non-remun (8)
Immigration	−0.040	0.020	−0.098	−0.027
	(0.020)	(0.030)	(0.060)	(0.022)
Baseline Mean	0.019	0.046	0.038	0.012
Observations	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. High Skilled workers are workers who completed secondary education. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Table B.7: Effects of Immigration on Entrepreneurship

	All	Natives	Migrants
IV - Price – Self-employed	(1)	(2)	(3)
Immigration	−0.039	−0.063	0.135
	(0.047)	(0.054)	(0.095)
Baseline Mean	0.119	0.121	0.111
Observations	3548	3548	3548
IV - Price – Small firms	(4)	(5)	(6)
Immigration	−0.027	−0.040	0.024
	(0.013)	(0.016)	(0.023)
Baseline Mean	0.013	0.013	0.011
Observations	3548	3548	3548
IV - Price – Big firms	(7)	(8)	(9)
Immigration	−0.013	−0.019	0.011
	(0.010)	(0.012)	(0.011)
Baseline Mean	0.006	0.006	0.005
Observations	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census. Big firms – six or more employees.

C Robustness

Table C.1: Effects of Immigration on Workers

	Wage employment			Log monthly wage		
	Overall (1)	Formal (2)	Informal (3)	Overall (4)	Formal (5)	Informal (6)
Panel A: IV-Price - Controlling for Lagged Outcome						
Observations	3548	3548	3548	3548	3467	3498
Panel B: IV-Price - Controlling for Lagged Population						
Immigration	0.189	0.472	−0.283	−1.204	−1.784	−1.290
	(0.104)	(0.167)	(0.100)	(0.408)	(0.523)	(0.509)
Observations	3548	3548	3548	3548	3548	3548
Panel C: IV-Price - Controlling for Lagged log(Migration)						
Immigration	0.106	0.405	−0.299	−1.263	−1.994	−1.458
	(0.111)	(0.166)	(0.113)	(0.459)	(0.619)	(0.594)
Observations	3548	3548	3548	3548	3548	3548
Panel D: IV-Price - Controlling for Lagged log(GDP)						
Immigration	0.120	0.424	−0.304	−1.226	−1.965	−1.363
	(0.109)	(0.166)	(0.111)	(0.428)	(0.580)	(0.551)
Observations	3548	3548	3548	3548	3548	3548
Panel E: IV-Price - Controlling for Lagged Industrial Composition						
Immigration	0.024	0.300	−0.277	−1.157	−1.809	−1.470
	(0.091)	(0.123)	(0.090)	(0.481)	(0.588)	(0.637)
Observations	3548	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Table C.2: Effects of Immigration on Firms

	Nb firms (1)	Entry rate (2)	Exit rate (3)	Nb jobs (4)	Firm wage (5)
Panel A: IV-Price - Controlling for Lagged Outcome					
Immigration	2.539	7.335	3.789	2.329	−3.388
	(0.575)	(1.987)	(0.922)	(0.802)	(1.144)
Observations	3548	3548	3548	3548	3548
Panel B: IV-Price - Controlling for Lagged Population					
Immigration	2.435	7.345	5.916	2.089	−2.984
	(0.646)	(2.086)	(2.065)	(0.882)	(1.020)
Observations	3548	3548	3548	3548	3548
Panel C: IV-Price - Controlling for Lagged log(Migration)					
Immigration	2.405	8.436	6.793	2.190	-2.969
	(0.690)	(2.529)	(2.512)	(0.932)	(1.027)
Observations	3548	3548	3548	3548	3548
Panel D: IV-Price - Controlling for Lagged log(GDP)					
Immigration	2.486	8.472	6.908	2.263	−2.887
	(0.683)	(2.443)	(2.476)	(0.907)	(0.971)
Observations	3548	3548	3548	3548	3548
Panel E: IV-Price - Controlling for Lagged Industrial Composition					
Immigration	1.945	6.595	5.167	2.155	−2.549
	(0.531)	(1.851)	(1.860)	(0.811)	(0.932)
Observations	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Table C.3: Effects of Immigration on Firms - Alternative Channels

	Nb firms (1)	Entry rate (2)	Exit rate (3)	Nb jobs (4)	Firm wage (5)
Panel A: IV-Price - Controlling for Local Price Shocks					
Immigration	2.171	7.044	4.482	2.492	−3.994
	(0.640)	(2.121)	(2.164)	(0.886)	(1.338)
Observations	3400	3400	3400	3400	3400
Panel B: IV-Price - Controlling for Capital Reallocation					
Immigration	2.455	7.307	4.915	2.515	−3.393
	(0.685)	(2.214)	(2.168)	(0.928)	(1.300)
Observations	2630	2630	2630	2630	2630
Panel C: IV-Price - Excluding Firms That Produce Agricultural Goods					
Immigration	2.807	7.488	5.641	2.381	−3.801
	(0.642)	(2.027)	(2.015)	(0.873)	(1.230)
Observations	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Droughts as the push shock

Construction of the drought shocks

An important determinant of rural to urban migration in Brazil are climate shocks, in particular the occurrence of droughts (Albert et al., 2021). Thus, to access the robustness of our main results, we also consider the incidence of droughts in the municipality of origin as a second push shock. We follow Bertoli et al. (2020); Albert et al. (2021) and use the SPEI (Standardized Precipitation-Evapotranspiration Index), which measures the water balance based on precipitation and evapo-transpiration due to temperature (Vicente-Serrano et al., 2010).[16] The data is available on a geo-referenced grid which we match to Brazilian municipalities.

We construct an indicator for drought which corresponds to negative values of the SPEI for each month m in each municipality of origin o (D_{om}). From the agricultural census, we also build an indicator g_{ocm} equal to one if the crop c is growing in municipality o in month m. Finally, we create the drought shock as the combination

[16]The data can be freely downloaded here https://spei.csic.es/home.html

Table C.4: Effects of Immigration on Workers - Controlling for Alternative Channels

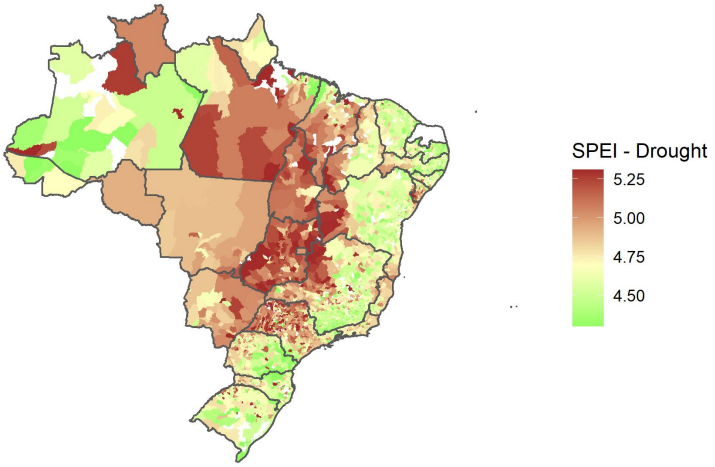
	Wage employment			Log monthly wage		
	Overall (1)	Formal (2)	Informal (3)	Overall (4)	Formal (5)	Informal (6)
Panel A: IV-Price - Controlling for Local Price Shocks						
Immigration	0.079	0.373	−0.294	−1.938	−2.455	−2.487
	(0.117)	(0.155)	(0.101)	(0.676)	(0.759)	(0.877)
Observations	3538	3538	3538	3538	3538	3538
Panel B: IV-Price - Controlling for Capital Reallocation						
Immigration	0.029	0.304	−0.275	−1.765	−2.283	−2.173
	(0.113)	(0.149)	(0.103)	(0.672)	(0.770)	(0.852)
Observations	2630	2630	2630	2630	2630	2630

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

of the drought indicator in the growing season of each group, with weights equal to the agricultural value the crop for the municipality. Figure C.1 shows the geographic distribution of the SPEI shock. Formally, the shock is given by:

$$s_o^{drought} = \sum_m \sum_c (\pi_{oc} \times g_{ocm} \times D_{om}) \tag{15}$$

Figure C.1: Droughts (SPEI measure)



Notes: The Drought shocks are constructed according to expression 15 in the text.

Results using Droughts as Push Shock

Table C.5: Effects of Immigration on Workers (2010)

	Wage employment			Log monthly wage		
	Overall (1)	Formal (2)	Informal (3)	Overall (4)	Formal (5)	Informal (6)
IV-Drought Immigration	−0.015	0.270	−0.285	−0.133	−0.677	−0.204
	(0.060)	(0.089)	(0.072)	(0.284)	(0.338)	(0.353)
F Statistic (IV)	18.05	18.05	18.05	18.05	18.05	18.05
Observations	3548	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Table C.6: Effects of Immigration on Firms (2011-2012)

	Nb firms (1)	Entry rate (2)	Exit rate (3)	Nb jobs (4)	Firm wage (5)
IV - Drought					
Immigration	1.634	2.564	2.782	2.037	−0.741
	(0.307)	(0.879)	(1.126)	(0.626)	(0.554)
F Statistic (IV)	18.05	18.05	18.05	18.05	18.05
Observations	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

D Effects on firm composition

Table D.7: Effects of Immigration on Shares by Sectors (2011-2012)

Panel A: IV-Price - Shares of Firms	Service	Construction	Manufacturing	Other Sectors
Immigration	0.232	0.174	−0.310	−0.097
	(0.149)	(0.052)	(0.135)	(0.134)
Baseline Mean	0.738	0.033	0.111	0.118
Observations	3548	3548	3548	3548

Panel B: IV-Price - Shares of Jobs	Service	Construction	Manufacturing	Other Sectors
Immigration	0.403	−0.132	−0.316	0.319
	(0.399)	(0.119)	(0.269)	(0.405)
Baseline Mean	0.465	0.041	0.185	0.309
Observations	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Table D.8: Effects of Immigration on Shares by Sizes (2011-2012)

Panel A: IV-Price - Shares of Firms					
	≤ 5	6 to 10	11 to 20	21 to 50	>50
Immigration	0.358	−0.165	−0.119	−0.079	0.005
	(0.135)	(0.067)	(0.047)	(0.040)	(0.034)
Baseline Mean	0.706	0.131	0.078	0.048	0.036
Observations	3548	3548	3548	3548	3548

Panel B: IV-Price - Shares of Jobs					
	≤ 5	6 to 10	11 to 20	21 to 50	>50
Immigration	0.049	−0.065	−0.134	−0.088	0.237
	(0.093)	(0.061)	(0.071)	(0.101)	(0.254)
Baseline Mean	0.129	0.079	0.086	0.112	0.594
Observations	3548	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

Table D.9: Effects of Immigration on Shares by Quartiles – Real Wages

Panel A: IV-Price - Shares of Firms				
	Q1	Q2	Q3	Q4
Immigration	−0.407	2.664	1.153	−3.410
	(0.617)	(0.958)	(1.223)	(1.439)
Observations	3548	3548	3548	3548

Panel B: IV-Price - Shares of Jobs				
	Q1	Q2	Q3	Q4
Immigration	−0.159	0.922	1.052	−1.814
	(0.304)	(0.427)	(0.721)	(1.046)
Observations	3548	3548	3548	3548

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census. Q1 is a bottom quartile, Q4 – top.

E Heterogeneous Effects

Table E.1: Heterogeneity by enforcement capacity

	Nb firms (1)	Entry rate (2)	Exit rate (3)	Nb jobs (4)	Firm wage (5)
Panel A: IV-Price - Controlling for Lagged Outcome					
Immigration	0.754	0.313	2.967	1.071	0.465
	(0.631)	(1.330)	(0.920)	(0.779)	(0.785)
Immigration*High Enforcement	2.454	9.469	1.010	1.676	−5.507
	(0.905)	(3.422)	(1.506)	(1.184)	(2.045)
High Enforcement	−0.174	−0.739	−0.127	−0.158	0.328
	(0.064)	(0.253)	(0.112)	(0.084)	(0.146)
Observations	3547	3547	3547	3547	3547
Panel B: IV-Price - Controlling for Lagged Population					
Immigration	0.824	0.439	1.497	1.077	0.460
	(0.620)	(1.321)	(1.323)	(0.764)	(0.746)
Immigration*High Enforcement	2.179	9.100	5.629	1.244	−4.873
	(0.958)	(3.326)	(3.405)	(1.197)	(1.785)
High Enforcement	−0.154	−0.713	−0.498	−0.124	0.283
	(0.068)	(0.245)	(0.265)	(0.086)	(0.127)
Observations	3547	3547	3547	3547	3547
Panel C: IV-Price - Controlling for Lagged log(GDP)					
Immigration	0.807	0.290	1.267	1.080	0.360
	(0.618)	(1.476)	(1.378)	(0.765)	(0.715)
Immigration*High Enforcement	2.336	11.182	7.608	1.432	−4.667
	(1.034)	(4.212)	(4.275)	(1.250)	(1.797)
High Enforcement	−0.160	−0.802	−0.564	−0.136	0.294
	(0.072)	(0.300)	(0.310)	(0.088)	(0.125)
Observations	3547	3547	3547	3547	3547

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census. High Enforcement – a dummy variable for the MCAs with a minimum distance to the labor office from the MCA's centroid over the median.

Table E.2: Heterogeneity by enforcement capacity – controlling for distance to state capitals

	Nb firms (1)	Entry rate (2)	Exit rate (3)	Nb jobs (4)	Firm wage (5)
IV-Price - Controlling for Distance to Capital					
Immigration	1.139	1.926	1.784	1.294	−1.239
	(0.830)	(1.712)	(1.018)	(0.994)	(1.594)
Immigration*High Enforcement	2.403	6.584	0.775	1.592	−6.026
	(1.155)	(2.683)	(1.371)	(1.231)	(3.120)
High Enforcement	−0.190	−0.525	−0.084	−0.144	0.418
	(0.084)	(0.197)	(0.104)	(0.090)	(0.233)
Observations	3450	3450	3450	3450	3450

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census. High Enforcement – a dummy variable for the MCAs with a minimum distance to the labor office from the MCA's centroid over the median.

Table E.3: Effects of Immigration by Terciles – GDP per capita

	Bottom (1)	Middle (2)	Top (3)
Panel A: Effects of Immigration on Workers (IV-price)			
Wage employment			
Overall	0.359	0.302	−0.320
	(0.118)	(0.188)	(0.240)
Formal	0.543	0.606	0.262
	(0.142)	(0.282)	(0.270)
Informal	−0.183	−0.303	−0.581
	(0.092)	(0.147)	(0.350)
Log monthly wage			
Overall	−0.939	−0.825	−2.819
	(0.501)	(0.477)	(1.895)
Formal	−1.530	−1.034	−3.437
	(0.570)	(0.511)	(2.237)
Informal	−1.051	−0.736	−3.893
	(0.620)	(0.661)	(2.518)
Panel B: Effects of Immigration on Firms (IV-price)			
Nb firms	2.809	2.345	1.452
	(0.786)	(0.863)	(1.016)
Entry	4.348	7.210	8.469
	(1.494)	(3.171)	(4.791)
Exit	3.046	6.342	4.734
	(1.601)	(3.666)	(2.545)
Nb jobs	1.652	3.125	1.395
	(0.957)	(1.089)	(1.496)
Firm wage	−3.007	−1.926	−6.293
	(1.072)	(1.142)	(4.386)
F Statistic (IV)	16.42	7.39	2.69
Observations	1171	1171	1206

Notes: Robust standard errors in parenthesis. All regressions control for the proportion of women, people younger than 18, and people that completed high-school at the destination municipality in the previous census. All regressions are weighted by the population in the destination municipality in the previous census.

F Model Appendix

This section contains the propositions and corollaries discussed in Section 4 and their respective proofs.

Proposition 1 ***(existence and uniqueness of the stationary equilibrium):*** *Under the assumptions that (i) the general law of motion of the productivity process in both sectors, denoted here by $G(\theta'|\theta)$, is continuous in both arguments and strictly decreasing in θ; and (ii) the profit functions are such that q_s and ℓ_s are continuous, single-valued and strictly increasing in θ (as in 4 and 5). Then, there exists a unique stationary competitive equilibrium where both the formal and informal sector exist, and both sectors have positive entry and exit.*

Proof: The proof of this proposition is organized in several steps.

Step 1: Properties of the profit and value functions

The assumption about the production and cost functions in both sectors implies that the profit functions are continuous and that q_j^* and l_j^* are continuous, single valued, and strictly increasing in θ. Write the corresponding equilibrium prices as

$$w^*(\mu) = W\Big(L^*(\mu, w(\mu))\Big) \tag{16}$$

It follows directly from Hopenhayn (1992), Lemma 3, that the function $w^*(\mu)$ is well defined and continuous. Using 16, one can re-write the value functions in each sector as follows:

$$V_f(\theta,\mu) = \tilde{\pi}_f(\theta,\mu) + (1-\delta_f)\beta \max\left\{0, E\left[V_f(\theta',\mu)\,\middle|\theta\right]\right\} \tag{17}$$

$$\begin{aligned} V_i(\theta,\mu) &= \tilde{\pi}_i(\theta,\mu) \\ &\quad +\beta \max\left\{0, (1-\delta_i)E\left[V_i(\theta',\mu)\,\middle|\theta\right], (1-\delta_f)E\left[V_f(\theta',\mu)\,\middle|\theta\right] - \tilde{c}^e\right\} \end{aligned} \tag{18}$$

where $\tilde{\pi}_s(\theta,\mu) \equiv \pi_s(\theta, w^*(\mu))$.

Lemma 1: *The functions $\tilde{\pi}_s$ are continuous in both arguments, strictly increasing in θ and decreasing in μ.*

Proof: The proof follows directly from the proof in Hopenhayn (1992). Continuity follows from the continuity of the profit functions in θ and w, and from continuity of $w^*(\mu)$. Since $p^*(\mu) > 0$, $\forall\mu$, it also follows from the properties of the profit function that $\tilde{\pi}_s$ is increasing in θ.

It remains to show that $\tilde{\pi}_j$ is decreasing in μ. Let $\mu_2 > \mu_1$ with the corresponding equilibrium prices w_1^* and w_2^*. Suppose by way of contradiction that $w_2^* < w_1^*$. This implies that $q_s(\theta, w_2^*) > q_s(\theta, w_1^*)$ $\forall\theta$ and therefore $Q_s^*(\mu_2) = \int q_s(\theta, w_2^*)\, d\mu_2^j(\theta) > \int q_s(\theta, w_1^*)\, d\mu_1^j(\theta) = Q_s^*(\mu_1)$ (the last inequality follows because q_j is strictly increasing in θ), $s = i, f$. But if $Q^*(\mu_2) > Q^*(\mu_1)$, then $L^*(\mu_2) > L^*(\mu_1)$ and therefore $w_2^* > w_1^*$ (as labor supply is fixed), which contradicts the initial assumption that $w_2^* < w_1^*$. Hence, if $\mu_2 > \mu_1$ then $w_2^* > w_1^*$. Hence, one can easily verify that $\tilde{\pi}_s(\theta, \mu_2) \equiv \tilde{\pi}_s(\theta, w_2^*) < \tilde{\pi}_s(\theta, w_1^*) \equiv \tilde{\pi}_s(\theta, \mu_1)$, which establishes the desired result. □

Given these properties of the profit functions, we can go ahead and establish the properties of the value functions:

LEMMA 2: *The functions, V_s, that solve 17 and 18 are unique and have the following properties: (i) they are continuous functions; (ii) strictly increasing in θ and decreasing in μ; and (iii) the option value in both sectors (the integral terms in 17 and 18) is strictly increasing in θ.*

Proof: Given the assumptions about the productivity process and the properties of $\tilde{\pi}_j$ established in Lemma 1, the properties (i) and (ii) follow directly from standard dynamic programming arguments. The same can be said about (iii), except that it also relies on (ii). Note that the presence of V_f in the option value of the informal firm does not alter the argument, as the value functions in both sectors share the same properties. □

Step 3: Uniqueness of the cut-offs for entry, exit and informal to formal transitions.

Due to the properties of the value functions established in Lemma 2 (continuity and monotonicity), it follows that the exit and entry thresholds in both sectors, $\underline{\theta}_s$ and $\underline{\nu}_s$, and the informal to formal transition cut-off, $\overline{\theta}_i$, are uniquely determined by the following expressions:

$$\int V_s(\theta', \mu)\, dF\left(\theta' \middle| \underline{\theta}_s\right) = 0, \qquad s = i, f \tag{19}$$

$$\int V_i(\theta, z)\, dF\left(\theta \middle| \underline{\nu}_i\right) = c_i^e \tag{20}$$

$$\int V_f(\theta, z)\, dF\left(\theta \middle| \underline{\nu}_f\right) = V_i^e\left(\underline{\nu}_f, w\right) - \left(c_i^e - c_f^e\right) \tag{21}$$

$$\int \left[V_f(\theta', \mu) - V_i(\theta', \mu)\right] dF\left(\theta' \middle| \overline{\theta}_i\right) = \tilde{c}^e \tag{22}$$

Using assumption (A.2.ii) and the functional form assumed for the production and cost functions in each sector, one can verify that $\underline{\theta}_f > \underline{\theta}_i$. This will be the case even

if at $\theta = \underline{\theta}_f$ the formal firm hires all of its employees informally. Using a similar reasoning, $c_f^e > c_i^e$ implies that $\underline{\nu}_f > \underline{\nu}_i$.

Finally, condition 22 requires a more careful explanation. The analysis of the static model showed that there is a productivity threshold $\overline{\theta}$ above which $\pi_f(\theta) > \pi_i(\theta)$, for any $\theta > \overline{\theta}$. Hence, from the results proved in Lemmas 1 and 2 it follows that, if there is informal to formal transition, then the threshold $\overline{\theta}_i$ defined by 22 is unique.

Step 4: The industry measures

Following Hopenhayn (1992), we define the operator $\hat{P}_k$ as follows:

$$\hat{P}_k \equiv \hat{P}_k(\theta, B) = \begin{cases} \int_B dF\left(x\middle|\theta\right) & \text{if} \quad \theta \in I_k \\ 0 & \text{otherwise} \end{cases}$$

for all Borel sets $B \subset \Theta$; where $I_1 = \left[\underline{\theta}_i, \overline{\theta}_i\right)$, $I_2 = \left[\overline{\theta}_i, \infty\right)$, and $I_3 = \left[\underline{\theta}_f, \infty\right)$. This is a bounded $\left(\left\|\hat{P}_k\right\| \leq 1\right)$, linear operator on the space of positive bounded measures: $\hat{P}_k\mu(B) = \int \hat{P}_k(\theta, B)\, d\mu(\theta)$, for all Borel sets $B \subset \Theta$ (see Hopenhayn, 1992). Using this notation, one can rewrite the law of motion of both sectors' measure as follows:

$$\mu_i' = \hat{P}_1\mu_i + \Lambda_i'\tilde{G}_i' \tag{23}$$
$$\mu_f' = \hat{P}_3\mu_f + \Lambda_f'\tilde{G}_f' + \hat{P}_2\mu_i \tag{24}$$

The properties of the productivity process and the definition of the sets I_k imply that $\left\|\hat{P}_k\right\| < 1$. Hence, the argument presented in Hopenhayn (1992, Lemma 4) follows directly and the operator $\left(\mathrm{I} - \hat{P}_k\right)$ has an inverse. Hence, the invariant measures μ_i and μ_f are well-defined and can be written as

$$\mu_i = \left(\mathrm{I} - \hat{P}_1\right)^{-1}\Lambda_i\tilde{G}_i \tag{25}$$
$$\mu_f = \left(\mathrm{I} - \hat{P}_3\right)^{-1}\left(\Lambda_f\tilde{G}_f + \hat{P}_2\mu_i\right) \tag{26}$$

Write the above measures as $\mu_i = m_i\left(\underline{\theta}_i, \overline{\theta}_i, \Lambda_i\right)$ and $\mu_f = m_f\left(\underline{\theta}_f\overline{\theta}_i, \Lambda_f\right)$, where we use the fact that the operators $\hat{P}_k$ are functions of the cut-offs that define the sets I_k. It is then useful to establish the following result:

Lemma 3: *The functions* $m_i\left(\underline{\theta}_i, \overline{\theta}_i, \Lambda_i\right)$ and $m_f\left(\underline{\theta}_f, \overline{\theta}_i, \Lambda_f\right)$ *are continuous in all of their arguments. Moreover,* $m_i\left(\underline{\theta}_i, \overline{\theta}_i, \Lambda_i\right)$ is *strictly increasing in* Λ_i *and* $m_f\left(\underline{\theta}_f, \overline{\theta}_i, \Lambda_f\right)$ is *strictly increasing in* Λ_f.

Proof: From expressions 25 and 26, it is clear that the functions $m_s(\cdot)$ are continuous and strictly increasing in Λ_s. The continuity in the cut-offs is a consequence of the fact that the operator $\hat{P}_k$ is continuous in the cut-offs that define the corresponding set I_k (see Hopenhayn, 1992, Lemma 5 for a proof).□

Step 5: Existence and uniqueness

The starting point of this final part of the proof is to use the following results established in Hopenhayn (1992): (i) there exists a stationary equilibrium with invariant measure μ, which is associated to an unique aggregate input-output pair, (L, Q), and unique prices (Theorem 2); (ii) if the entry cost is low enough, a stationary equilibrium with positive entry exists (Theorem 3); (iii) if the profit function is multiplicatively separable between productivity and prices, $\pi(\theta, p, w) = h(\theta)\, g(p, w)$, then if there exists an equilibrium with entry and exit, it is unique (Theorem 4).[17] Given these results, it remains to show that there exists an unique stationary equilibrium with an unique formal-informal partition and entry into both sectors.

First, write $\mu = m\left(\underline{\theta}_i, \overline{\theta}_i, \underline{\theta}_f, \Lambda_f, \Lambda_i\right) = m_i\left(\underline{\theta}_i, \overline{\theta}_i, \Lambda_i\right) + m_f\left(\underline{\theta}_f, \overline{\theta}_i, \Lambda_f\right)$. Fix μ; from Step 3 (equations 19-22) we know that there are unique cut-off points for entry, exit and transition between sectors. In particular, because $c_f^e > c_i^e$ the following ordering between formal and informal entry thresholds holds: $\underline{\nu}_f > \underline{\nu}_i$. Thus, for any $\nu \in [\underline{\nu}_i, \underline{\nu}_f)$, entry occurs into the informal sector and for any $\nu \geq \underline{\nu}_f$, entry occurs into the formal sector. Hence, the unique thresholds $\left(\underline{\nu}_i, \underline{\nu}_f\right)$ pin down the mass of entrants into the informal sector, Λ_i. Additionally, fixing μ uniquely determines thresholds $\left(\underline{\theta}_i, \overline{\theta}_i\right)$, which therefore pins down a unique informal sector size, $\mu_i = m_i\left(\underline{\theta}_i, \overline{\theta}_i, \Lambda_i\right)$.

Finally, the industry size μ also uniquely determines the formal sector's threshold, $\underline{\theta}_f$. But $\mu = \mu_i + \mu_f$ and $\mu_f = m_f\left(\underline{\theta}_f, \overline{\theta}_i, \Lambda_f\right)$ is strictly increasing in Λ_f. Thus, once the informal sector size is determined and the thresholds $\left(\underline{\theta}_f, \overline{\theta}_i\right)$ are fixed, there is an unique value of Λ^f that satisfies the identity $\mu = \mu_i + \mu_f$.

Therefore, there is an unique stationary equilibrium with invariant sector measures, μ_s, entry and exit thresholds, $\left(\underline{\nu}_s, \underline{\theta}_s, \overline{\theta}_i\right)$, $s = i, f$, aggregate prices and quantities, (Q, L, w), and entry levels in both sectors, (Λ_f, Λ_i).

□

In what follows we focus on the analysis of the model's implications regarding the behavior of some key outcome variables over firms' life cycle under the stationary equilibrium.

[17] The functional form assumed for the production function belongs to this class and thus the theorem applies directly.

The life cycle of firms: size, productivity and informality

Let the productivity distribution among firms of age n in sector s be denoted by the probability measure λ_s^n. This measure is defined over the set of active firms with a given age, which is denoted by A_s^n. The evolution of the productivity distribution in the informal and formal sectors within a given cohort is given by:

$$\begin{aligned}\lambda_i^{n+1}(\theta') &= \int_{X_1} G\left(\theta'\middle|\theta\right) d\lambda_i^n(\theta) \\ \lambda_f^{n+1}(\theta') &= \int_{X_2} G\left(\theta'\middle|\theta\right) d\lambda_f^n(\theta) + \int_{X_3} G\left(\theta'\middle|\theta\right) d\lambda_i^n(\theta)\end{aligned}$$

for all $\theta' \in \Theta$; $X_1 \equiv A_i^n \cap \left[\underline{\theta}_i, \overline{\theta}_i\right)$, $X_2 \equiv A_f^n \cap \left[\underline{\theta}_f, \infty\right)$, and , $X_3 \equiv A_i^n \cap \left[\overline{\theta}_i, \infty\right)$.

Now consider the ordering of measures given by the the first order stochastic dominance criterion, so that $\lambda_s^{n+1} \succeq \lambda_s^n$ means that the productivity distribution at age $n+1$ first order stochastically dominates the one at age n. Then the following result holds:

Proposition 2: *Assume that the assumptions of Proposition 1 hold and that the pre-entry signal parameter has a continuous distribution, $H(\nu)$. Then the productivity distribution among active firms within a sector and a cohort is increasing in the cohort's age. That is, $\lambda_s^{n+1} \succeq \lambda_s^n$ for all n and $s = i, f$.*

Proof:

Define the following operators:

$$T_k(B) = \begin{cases} \int_B dF\left(x\middle|\theta\right), & \text{if } \theta \in X_k \\ 0 & \text{otherwise} \end{cases}$$

for $k = 1, 2, 3$ and all Borel sets $B \subseteq \Theta$. As before, $X_1 \equiv A_i^n \cap \left[\underline{\theta}_i, \overline{\theta}_i\right)$, $X_2 \equiv A_f^n \cap \left[\underline{\theta}_f, \infty\right)$, and , $X_3 \equiv A_i^n \cap \left[\overline{\theta}_i, \infty\right)$. One can then rewrite the expressions for the λ_s^n as follows:

$$\begin{aligned}\lambda_i^{n+1} &= T_1 \lambda_i^n \\ \lambda_f^{n+1} &= T_2 \lambda_f^n + T_3 \lambda_i^n\end{aligned}$$

The productivity distribution of newborn firms, however, is simply given by $\lambda_i^1([\theta_l, \infty)) = \int_{\nu > \underline{\nu}_i} F\left(\theta\middle|\nu\right) dG(\nu)$ and $\lambda_f^1([\theta_l, \theta)) = \int_{\nu > \underline{\nu}_f} F\left(\theta\middle|\nu\right) dG(\nu)$.

From the second period on, the productivity distribution in every subsequent period is obtained by first applying the truncation implied by the conditions $\theta \in X_k$ in the operators T_k. That is, the operators T_k not only condition on the "stayers" region of the informal and formal sectors, but on the intersection of these regions

and the set of active firms. This always implies a truncation on the lower tail of the productivity distribution and hence this truncation is increasing in the FOSD ordering (the truncated distribution FOSD the unrestricted distribution). Finally, assumption (A.2) implies that after the truncation, the operator T_k is also monotone and hence the operator T_k as defined is increasing in the FOSD criterion. Thus, the following holds almost by definition: $\lambda_i^2 \equiv T_1\lambda_i^1 \succeq \lambda_i^1$. By induction, $\lambda_n^{n+1} \succeq \lambda_i^n$ for any n.

The analysis of λ_f^n is not so straightforward because of the presence of the term $T_3\lambda_i^n$, so a more careful argument is needed. First, note that because of the argument just made for λ_i^n, in the absence of the term $T_3\lambda_i^n$ one would observe $\lambda_f^{n+1} \succeq \lambda_f^n$ for any n. Second, the conditioning embedded in operator T_3 is stronger than the one in T_2, as $\overline{\theta}_i > \underline{\theta}_f$. Put differently, the productivity distribution of those that make the informal to formal transition is truncated on the left at a higher point than the distribution among the formal "stayer" firms. Thus, $\lambda_f^2 = T_2\lambda_f^1 + T_3\lambda_i^1 \succeq \lambda_f^1$. But given the result that λ_i^n is increasing in n, and that the T_k operators are increasing, then λ_f^n will be also increasing in n. □

Corollary 1: *As the productivity distribution among active firms within a sector and a cohort is increasing in the cohort's age, so is the integral of any function that is increasing in θ. In particular, this is true for the survival rate, average size and average revenue.*

Proof: Given the result in Proposition 2, this corollary follows mechanically and no proof is provided.

Corollary 2 (intensive margin of informality): *As a consequence of Proposition 2 and Corollary 1, the average informality rate within formal firms in a given cohort is decreasing in cohort's age.*

Proof:

As discussed in Section 4, there is an unique threshold $\tilde{\ell}$ above which the formal firm only hires formal workers (on the margin). Thus, if the firm grows in size above $\tilde{l}$, all the workers in excess of $\tilde{\ell}$ will be hired formally. For a formal firm that has its optimal level of labor below the threshold, $\ell_f^*(\theta) < \tilde{\ell}$, $s_i(\theta) = 1$. In this case, the within informality rate will stay constant at one for some period while the firm is growing, but as soon as $\ell_f^*(\theta) = \tilde{\ell}$ the informal share will start declining monotonically with firm's size. Similarly, for any initial value of $s_i < 1$, as the firm grows the s_i will decline monotonically. Hence, the s_i is a constant function of θ if $\ell(\theta) < \tilde{\ell}(\theta)$ and it is strictly decreasing in θ over the range where $\ell(\theta) \geq \tilde{\ell}(\theta)$. Combined with the result in Proposition 2, this implies that the average within firm informality for a given cohort will be monotonically decreasing with cohort's age almost everywhere in the relevant range $\left[\underline{\theta}_f, \infty\right]$. In fact, as long as there is always at least one firm within the cohort that has a $s_i(\theta) < 1$, the average within firm informality will be strictly decreasing in the cohort's age. □

It is straightforward to see that the share of informal workers is decreasing in firm's size (and therefore productivity) in the within-period (static) problem of the firm. Thus, by Proposition 1 and Corollary 1 one gets Corollary 2. Finally, let the informality rate among firms of a given cohort of age n be expressed as $B^n = \frac{\mu\left(A_i^n\right)}{\mu\left(A_i^n\right)+\mu\left(A_f^n\right)}$, where $\mu\left(A_s^n\right)$ denotes the measure of active firms in sector s with age n. With this notation in hand, the last result of this section can be stated as follows:

Corollary 3 (extensive margin of informality) *Under the conditions of Proposition 2, the informality rate B^n within a given cohort is weakly decreasing in cohort's age. That is, $B^{n+1} \leq B^n$ for all n.*

Proof:

Proposition 2 established that the distribution of productivity is increasing with firms' age in both sectors. At the same time, the informal to formal transition threshold, $\overline{\theta}_i$ remains constant. Hence, as the cohort gets older a smaller number of firms (among the survivors) will remain informal as the most productive ones keep making the transition into the formal sector. This implies that $\mu_i^{n+1} \leq \mu_i^n$. The same is not true for μ_f^n, as there is no upper limit for formal stayers. This means that formal firms do not face the additional exit margin that informal firms do, as there is no upper limit for their growth. Since the productivity shock in both sectors is the same, even if the set of active firms in the formal sector reduces in size as the cohort ages, it does so at a lower rate than informal firms in the same cohort. Hence, the ratio $\gamma_b^n = \frac{\mu_i^n}{\mu_i^n+\mu_f^n}$ is weakly decreasing in the cohort's age. □

G Model Estimation Appendix

G.1 Estimating formal sector's persistence parameter

One can use the first order condition of formal firms that hire at least one formal worker and the law of motion of formal firms' log-productivity to show that the establishment-level employment process can be represented as a simple AR(1) process with the same persistence parameter, ρ_f. This procedure assumes that there are no adjustment costs, such as hiring costs. This could in principle lead to an overestimation of the persistence parameter ρ_f, as it would be also capturing these frictions and not only the persistence of the productivity process *per se*. However, Dix-Carneiro et al. (2021) estimate this persistence parameter in a richer model with adjustment costs and find slightly higher (and not lower) values.

Let the log of firm j's employment at time t be denoted by $n_{j,t}^f = \ln\left(\ell_{j,t}\right)$; using formal firms' first order condition and the law of motion for the log-productivity (7), one can write

$$n_{j,t}^f = \gamma_0 + \gamma_1 \log\left(\theta_{j,t}\right) - \gamma_2 w_t + m_{j,t} \tag{27}$$

where w_t denotes wages, $\gamma_0 = \frac{1}{1-\alpha}\left[\log(\alpha) + \log\left(\frac{1-\tau_y}{1+\tau_w}\right)\right]$, $\gamma_1 = \frac{1}{1-\alpha}$, and $m_{j,t}$ is measurement error.

One can then use (8) to write (27) in its dynamic representation:

$$n^f_{j,t} = b_0 - \gamma_1 w_t + b_2 w_{t-1} + \rho_f n^f_{j,t-1} + \eta_j + e_{j,t} \tag{28}$$

where $\eta_j \equiv \gamma_1 (1-\rho_f) \ln \nu_j$, $b_0 \equiv (1-\rho_f)\gamma_0$, $b_2 \equiv \gamma_1 \rho_f$, and the error term is given by $e_{j,t} = \gamma_1 \epsilon_{j,t} + m_{j,t} - \rho_f m_{j,t-1}$.

The employment process can be represented as a simple AR(1) process with an MA(1) error, where the MA(1) component arises if one allows for measurement error. If there is no measurement error in (27), then the error term in the above expression, $e_{j,t}$, is serially uncorrelated. Thus, it is possible to estimate the persistence parameter of the productivity process from the employment process of formal firms. The final regression estimated is the following:

$$n^s_{j,t} = \rho_f n^f_{j,t-1} + \Gamma \mathbf{X}_{j,t} + \eta_j + e_{j,t}, \tag{29}$$

where $\mathbf{X}_{j,t}$ denotes a vector of controls in addition of $n^f_{j,t-1}$ and η_j denotes firm's fixed effect. The vector $\mathbf{X}_{j,t}$ includes a set of year dummies and the current and lagged log-average wage rate calculated at the 4-digit industry level. This specification, including the inclusion of current and lagged wages, is standard in the empirical literature (e.g. Arellano and Bond, 1991; Blundell and Bond, 1998).

We start by estimating (29) using both a standard OLS estimator and a within-groups estimator. The former is known to be upward biased while the latter is downward biased, and they can thus be used as reference upper and lower bounds to any consistent estimator. The third model used is the standard first-differenced GMM estimator discussed in Arellano and Bond (1991). However, this estimator can be subject to severe finite sample biases (towards zero) when the lagged levels are weak instruments for the first-differenced equation. This will be the case when the series are very persistent ($\rho_f \to 1$) or when $\frac{\text{var}(\eta_j)}{\text{var}(e_{j,t})}$ is high (see Blundell and Bond, 1998, for a detailed analysis). The forth model is the system GMM estimator (Blundell and Bond, 1998), which uses lagged differences as instruments for equations in levels.[18]

For the first-differenced and system GMM models, we consider two scenarios for the error term. The first is the assumption of no measurement error, which allows for the use of lagged levels dated $t-2$ and earlier as instruments for the differenced equations, and lagged differences dated $t-1$ and earlier for the level equations. The second case assumes that there is measurement error, which implies that the error term in (29) will have a MA(1) structure. I this case one can only use lagged levels

[18]The validity of this additional set of moment conditions relies on a relatively mild stationarity assumption regarding the initial conditions of the process (for a discussion, see Blundell and Bond, 1998), which is satisfied under the maintained assumptions of the model.

dated $t-3$ (and earlier) and lagged differences dated $t-2$ and earlier as instruments. All GMM regressions consider the log-wage as a predetermined variable and they are estimated using the two-step estimator with the correction for the variance-covariance suggested by Windmeijer (2005). Table G.4 shows the results.

Table G.4: Productivity process estimation

Dep. Var.: Log-Employment ($n_{j,t}$)						
	OLS	FE	DIFF1	DIFF2	SYS1	SYS2
$n_{j,t-1}$	0.944**	0.497**	0.594**	0.728**	0.713**	0.921**
	(0.000)	(0.002)	(0.007)	(0.011)	(0.009)	(0.005)
$\log(w_t)$	0.0030	0.0100	-0.0920	-0.158*	-0.210**	-0.339**
	(0.006)	(0.008)	(0.066)	(0.062)	(0.072)	(0.069)
$\log(w_{t-1})$	0.006	0.005	0.069	0.054	0.110	0.075
	(0.006)	(0.007)	(0.095)	(0.048)	(0.069)	(0.053)
Obs.	741,268	741,268	741,268	741,268	741,268	741,268

Notes: DIFF1 and DIFF2 are the difference GMM models without and with measurement error; SYS1 and SYS2 are the system GMM models without and with measurement error, respectively. Significant at ***1%, **5% and *10% levels.

The results shown in Table G.4 follow the pattern expected from the standard results in the literature. The OLS result already points to a very persistent series, which is confirmed by the downward bias apparent in the first-differenced GMM model (DIFF) as compared to the system GMM estimator (SYS). Blundell and Bond (1998) find similar results when comparing these two estimators using a small sample of British firms. There is also evidence that measurement error is indeed present, as the estimates under the assumption of no measurement error (DIFF1 and SYS1) seem to be substantially downward biased. The Sargan tests for the additional instruments available in the DIFF1 and SYS1 models (relatively to the DIFF2 and SYS2, respectively) strongly reject the validity of these additional instruments, reinforcing the evidence of measurement error (results not reported). That said, the preferred model is the system GMM under the assumption of measurement error (SYS2), which provides a reasonable value for the persistence parameter.[19]

[19]The Sargan test of overidentifying restrictions rejects instruments' validity in all models, with p-values of zero (not reported). However, Arellano and Bond (1991) show in their simulations exercise that the Sargan test rejects too often in the presence of heteroskedasticity, which is confirmed in their empirical application (they estimate a dynamic employment equation very similar to (29).

G.2 SMM: Implementation Details

Let the vector of simulated moments is denoted by $m_S(\Omega;\zeta)$, where ζ denotes the vector of parameters determined in the first step. The estimator is given by

$$\hat{\Omega} = \arg\min_{\Omega} \left(\hat{m}_N - m_S(\Omega;\zeta)\right)' \hat{\mathbf{W}} \left(\hat{m}_N - m_S(\Omega;\zeta)\right) \tag{30}$$

which is asymptotically normal. The $\hat{\mathbf{W}}$ is the optimal weighting matrix, which is the positive, semi-definite matrix that minimizes the asymptotic variance.

The estimation procedure takes the observed wages as given. In order to obtain a wage measure that is more consistent with the model, we estimate a log-wage regression controlling for schooling, gender (male dummy), 4-digit industry dummies, state of residence, a dummy for whether the worker holds a formal contract, dummy for whites, age and age squared, tenure in current job and tenure squared. We restrict the sample to employees only (formal or informal), who are 18 to 69 years old, and who have worked at least 20 hours but at most 84 hours (the 99th percentile) in a given week. The lower bound of 20 hours seeks to exclude those who are still in school or with very low attachment to the labor market. We use the estimated coefficients to compute the adjusted wage evaluated at the mean of the vector of observables.

For the estimation, we use a cohort of $500,000$ potential entrants, which we follow until the age of 50 (within this class of models, Sterk et al. (2021) use a cohort of size $100,000$, which they follow for 20 years). For each potential entrant, we draw a pre-entry productivity parameter (ν_j) and a sequence of post entry productivity shocks, $\epsilon_{j,t}$ for $j = 1, ..., 50$. The stochastic components of the model are drawn only once in the beginning of the procedure and are kept fixed during the estimation. Each potential entrant has an individual pre-entry productivity parameter, ν_j, which is a firm-specific intercept in the AR(1) process that the firm faces after entry. Hence, for each value of ν_j there is a specific transition matrix between states. To save on computational time, we use 101 equally spaced grid points for the productivity space. We compute a separate transition probability matrix for each point in the grid and also for the formal and informal sectors (which have different persistence, ρ_s, and shock variance, σ_s^2). We do so using the method proposed by Tauchen (1986).

Smooth Policy Functions

Some of the main decisions firms make in this model are discrete: whether to enter the formal or informal sector; if active, to exit or not; and if informal, to become formal or not. This implies that some of the policy functions will be step functions of the parameter vector, β. This imposes some challenges to the estimation, as these discontinuities are transmitted to the estimator via the objective function described in 30. In particular, is not possible to use derivative-based methods, which are faster and more accurate than derivative-free methods or random search algorithms. We

therefore use the smoothing function proposed by Bruins et al. (2015) to correct for the choppiness of the policy functions:

$$h\left(\tilde{V}(\beta), c, \lambda\right) = \frac{\tilde{V}_c(\beta)/\lambda}{1 + \sum_k \tilde{V}_k(\beta)/\lambda}$$

where $\tilde{V}(\beta)$ is the set of payoffs associated to firms' choices, such as whether to transit to the formal sector or not (if the firm is informal). $\tilde{V}_c(\beta)$ denotes the net payoff of a given choice c and λ denotes the smoothing parameter. As $\lambda \to 0$, $h(\cdot)$ goes to one if the alternative c provides the highest payoff and zero otherwise. There is a trade-off between bias and smoothness in the choice of the smoothing parameter: a large value for λ provides a smoother objective function, but can lead to biased estimates; a small value reduces bias but increases choppiness (Keane and Smith, 2003). This latter effect can be alleviated by choosing a large number of simulations. We follow Altonji et al. (2013) and choose $\lambda = 0.05$.

Standard Errors

The conditions for consistency are almost the same as in the standard Generalized Method of Moments (GMM) estimator. The main difference lies on the regularity conditions required to guarantee consistency. As in Ulyssea (2018), we use the conditions for both weak and strong consistency provided by Duffie and Singleton (1993), which are satisfied here. The following assumptions are required for asymptotic normality to hold: (i) β_0 (the true parameter vector) and $\hat{\beta}$ are interior to the parameter space; (ii) the simulator used to generate the simulated data is continuously differentiable w.r.t. β in a neighborhood $\mathcal{B}$ of β_0; and (iii) $\mathbf{G}_0 \equiv E\left[\nabla_\beta g_{NS}\left(\beta_0;\zeta\right)\right]$ exists, is finite and $\mathbf{G}_0'\mathbf{W}\mathbf{G}_0$ is nonsingular, where $g_{NS}\left(\beta_0;\zeta\right) = \hat{m}_N - m_S(\beta_0;\zeta)$ and $\mathbf{W}$ is a positive semi-definite matrix.

The derivation of the asymptotic distribution is standard and implies the following:

$$\sqrt{N}\left(\hat{\beta} - \beta_0\right) \xrightarrow{d} N\left(0, \left(\mathbf{G}_0'\mathbf{W}\mathbf{G}_0\right)^{-1}\mathbf{G}_0'\mathbf{W}\Sigma_s\mathbf{W}\mathbf{G}_0\left(\mathbf{G}_0'\mathbf{W}\mathbf{G}_0\right)^{-1}\right)$$

where $\Sigma_s = \kappa\Sigma$, $\kappa = \lim_{N\to\infty}\left(1 + \frac{N}{S(N)}\right)$ and $\Sigma = E\left[g\left(\beta_0\right)g\left(\beta_0\right)'\right]$ is the GMM asymptotic variance-covariance matrix; N is the number of observations used to obtain the vector of moments, and $S(N)$ is the size of the cohort of firms simulated, which is an increasing function of the number of observations, such $\frac{N}{S(N)}$ converges to a constant (e.g. Gourinchas and Parker, 2002).

Analogously to GMM, the optimal weighting matrix is given by $W^* = \Sigma_s^{-1}$, which reduces the asymptotic variance-covariance matrix to

$$V_s\left(W^*\right) = \left(\mathbf{G}_0'\mathbf{\Sigma_s^{-1}}\mathbf{G}_0\right)^{-1}$$

The variance-covariance is computed using the empirical counterpart of Σ, which is given by a diagonal matrix with the empirical variances of the moments. Following Adda et al. (2017), we use robust standard errors to compute the variances of moments obtained from basic regressions and we use a bootstrap method with 500 replications for the remaining moments. Using bootstrap and the estimated standard errors from regressions deliver essentially the same results. We use standard numerical differentiation methods to obtain G_0.

G.3 Identification discussion

Following Adda et al. (2017), we start by evaluating how convex the objective function is at the area around the estimated vector of parameters. The intuition of the exercise is simple: if the model is (locally) identified, the objective function should have curvature in the region around the estimated vector of parameters, $\hat{\beta}$. If the objective function was to be found flat around the estimated vector of parameters, it would be an indication that the moments used do not provide relevant information to identify the parameters.

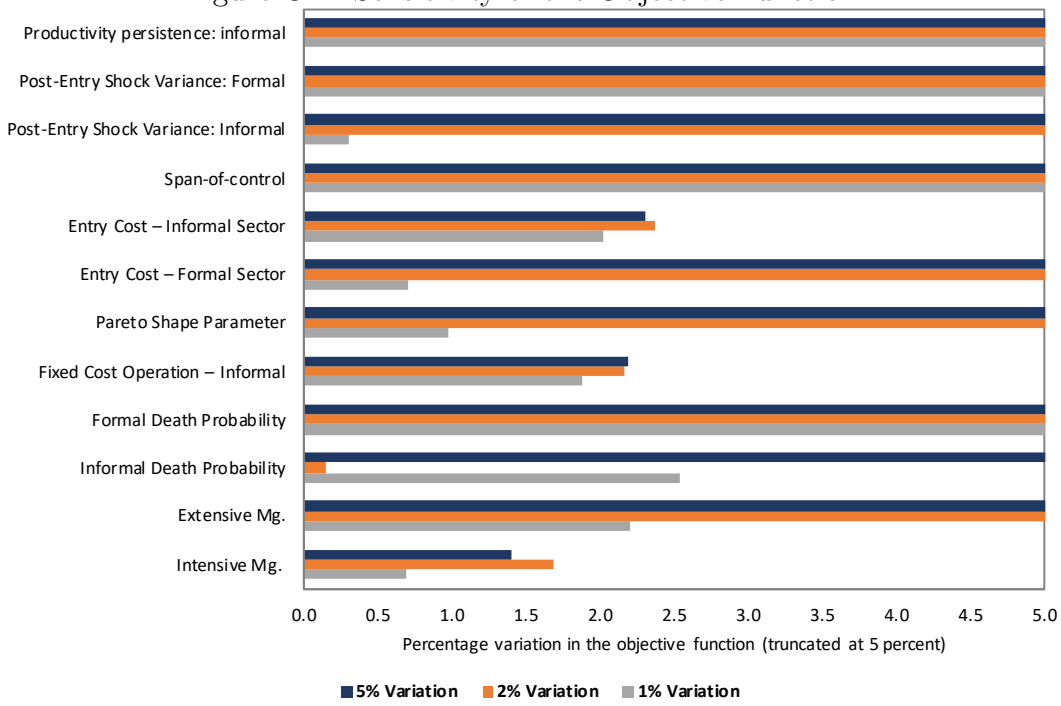
To do that, we re-compute the objective function varying each parameter by 1, 2 and 5 percent from its estimated value and compute the variation in the objective function relative to its original value (evaluated at $\hat{\beta}$). To report the results, we plot the percentage change in the objective function that corresponds to the perturbation in each estimated parameter. Figure G.1 shows that there is substantial variation in the objective function in response to small perturbations of the different parameters.

Moments' sensitivity to different parameters

Here we start by discussing what variation in the data allows us to pin down different parameters. The cost function parameters for both margins of informality are identified by the variation in the share of informal firms by firm size (for the parameter φ_i) and by the behavior of the share of informal workers within formal firms by firm size (for the parameter φ_f). In the model, the cost functions of both margins are increasing in firms' size, which implies that the average share of informal firms and the average share of informal workers within formal firms will be decreasing in firm size, as observed in the data. The intensity of this negative relationship in the model is governed by φ_i and φ_f and therefore the behavior of these moments in the data provides the relevant information to estimate these parameters.

The death shocks in both sectors determine the relative disadvantage of being informal regardless of firms' productivity, size or age: the higher the informal death shock, δ_i (relative to the formal sector's, δ_f), the greater the relative disadvantage of being informal. They are therefore disciplined by the different moments that speak to the relative size of the informal sector. Importantly, the estimate of the exogenous exit probability in the formal sector is consistent with the exit probability observed

Figure G.1: Sensitivity of the Objective Function



Note: The horizontal bars show the percentage change in the objective function with respect to one, two and five percent changes in the given parameter of the model. The figure is truncated on the right at 5 percent.

in the data among older (and typically larger) formal firms.[20] This is reassuring, as older firms (both in the model and in the data) are more stable and less likely to endogenously exit due to a very negative productivity shock. Thus, the exit rates among older firms seem like a reasonable approximation for the formal death shock, δ_f. The entry costs into both sectors are also directly linked to the relative size of the informal sector, but more importantly they directly affect the left tail of the size distribution in both sectors. If c_f^e is too high, there will be too few small formal firms and the size distribution will be more shifted to the right. Hence, the moments of firm size distribution in both sectors help pinning down the entry costs.

The shape parameter of the Pareto distribution, ξ, is largely determined by the moments of firm size distribution in the formal and informal sectors, and in particular the skewness in formal sector's distribution. The informal persistence and the variance of the shock of the productivity processes in both sectors – ρ_i and σ_s^2, $s = i, f$ – are linked to the moments related to firm growth in both sectors. Also importantly,

[20]Using a simple linear probability regression conditioning on year and industry dummies, the predicted exit rate at ages 6 to 10 monotonically declines from 0.074 to 0.038.

the σ_s are directly connected to the degree of overlap between formal and informal firm size distributions, as they determine how much productivity dispersion there is conditional to sector choice. Put differently, if there was no uncertainty not only there would be no firm growth, but also size distributions in both sectors would be disjoint. These sources of variation in the data are crucial to separately estimate the variances and exogenous death shocks.

Finally, we quantitatively investigate the issue of which moments are more directly connected to certain parameters. The goal is to complement the above sensitivity discussion and provide direct evidence of which moments (computed from simulated micro data) are more sensitive to some key estimated parameters. For that, we use the estimated vector of parameters and change one parameter at a time, simulate the model to generate new micro data and then recompute the main moments used in the estimation. In particular, we focus on the main non-policy parameters that are estimated via SMM in the second step, namely: the formal and informal productivity processes variances of the shocks, σ_s^2; and the exogenous exit probabilities, δ_s, $s = i, f$.

Table G.5: Moments' sensitivity to formal sectors' productivity shock SD (σ_f)

	Baseline $\sigma_f = 0.148$	$\sigma_f =$ 0.05	0.10	0.15	0.20	0.25
Share Informal workers	0.298	0.271	0.253	0.316	0.285	0.343
Share Informal Firms	0.696	0.716	0.696	0.718	0.446	0.126
Intensive Mg. (% informal in formal firms)	0.412	0.420	0.405	0.462	0.637	0.800
Informal Firms Size Distribution (in %)						
≤ 2 employees	0.957	0.989	0.973	0.927	0.882	0.873
≤ 5 employees	0.998	1.000	1.000	1.000	0.995	0.990
Formal Firms Size Distribution (in %)						
≤ 5 employees	0.697	0.668	0.642	0.696	0.833	0.930
6 to 10 employees	0.144	0.158	0.161	0.133	0.077	0.036
11 to 20 employees	0.083	0.098	0.109	0.092	0.048	0.019
21 to 50 employees	0.048	0.053	0.061	0.055	0.029	0.011
> 50 employees	0.028	0.023	0.027	0.024	0.013	0.005
Formal firms growth						
$Size_{age=5}/Size_{age=1}$	1.302	0.945	1.113	1.317	1.377	1.493
$Size_{age=10}/Size_{age=1}$	1.518	0.824	1.098	1.499	1.587	1.852
Informal firms growth						
$Size_{age=5}/Size_{age=1}$	1.064	0.993	1.026	1.054	1.073	1.094

Table G.6: Moments' sensitivity to informal sectors' productivity shock SD (σ_i)

	Baseline $\sigma_i = 0.144$	0.05	0.10	$\sigma_i =$ 0.15	0.20	0.25
Share Informal workers	0.298	0.312	0.270	0.270	0.179	0.114
Share Informal Firms	0.696	0.388	0.543	0.679	0.578	0.545
Intensive Mg. (% informal in formal firms)	0.412	0.534	0.433	0.447	0.498	0.499
Informal Firms Size Distribution (in %)						
≤ 2 employees	0.957	0.892	0.898	0.932	0.959	0.971
≤ 5 employees	0.998	0.999	0.999	1.000	1.000	1.000
Formal Firms Size Distribution (in %)						
≤ 5 employees	0.697	0.861	0.736	0.677	0.680	0.650
6 to 10 employees	0.144	0.075	0.130	0.139	0.122	0.111
11 to 20 employees	0.083	0.037	0.075	0.098	0.096	0.098
21 to 50 employees	0.048	0.019	0.040	0.060	0.067	0.083
> 50 employees	0.028	0.008	0.018	0.026	0.034	0.058
Formal firms growth						
$Size_{age=5}/Size_{age=1}$	1.302	0.973	1.151	1.297	1.388	1.239
$Size_{age=10}/Size_{age=1}$	1.518	0.926	1.217	1.428	1.504	1.292
Informal firms growth						
$Size_{age=5}/Size_{age=1}$	1.064	1.072	1.069	1.047	1.024	0.993